

Homogeneous Cadence Redundancy in a Weak-Lensing Forward Model

A consistency test of the RM CNL–2D branch using Conicus3D as an external benchmark

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Abstract. The weak-lensing forward model used by Zhao, Scognamiglio, Doré and Bouman in *Generative Diffusion Priors for 3D Mapping of the Dark Universe* provides a particularly clean external benchmark because it separates a physical projection operator from the statistical prior used for three-dimensional mass reconstruction. This note asks a narrow question: what part of that forward operator is already reproduced by the current local Relativity of Matter (RM) branch, and what part remains genuinely underived? Within the frozen CNL–2D/Pure-Cadence branch, the static isotropic optical solution reproduces the standard scalar weak-lensing response on its stated domain. More importantly, a homogeneous cadence factor $N_0(T)$ is shown to be exactly removable from the local gravity-plus-matter action by the reparametrisation $d\bar{T} = N_0(T)dT$. Therefore N_0 is not an independent local cosmological function; the physically nontrivial lapse/cadence question is carried by the inhomogeneous perturbation $\nu = \delta N/N_0$. The Kaiser–Squires convolution survives kinematically for a scalar refractive index, while the full cosmological projection kernel remains open because the RM background production law, the perturbative lapse equation and the growth history have not yet been derived. No private COSMOS-Web product is used, and no observational validation of RM is claimed.

External benchmark: the Conicus3D forward model

Zhao *et al.* write the weak-lensing convergence for a source at comoving distance w as

$$\kappa(\boldsymbol{\theta}, w) = \mathbf{Q}\delta = \frac{3H_0^2\Omega_m}{2c^2} \int_0^w dw' \frac{w'(w-w')}{w} \frac{\delta(w'\boldsymbol{\theta}, w')}{a(w')}, \quad (1)$$

then obtain the shear through the usual two-dimensional convolution

$$\gamma = \mathbf{P}\kappa, \quad (2)$$

and finally use

$$e_{\text{obs}} = \mathbf{P}\mathbf{Q}\delta + \varepsilon. \quad (3)$$

The supplement connects Eq. (1) to the comoving Poisson equation

$$\Delta\Phi = \frac{3H_0^2\Omega_m}{2a} \delta. \quad (4)$$

The useful separation for the present purpose is therefore

$$\boxed{\text{local optical response}} \longrightarrow \boxed{\text{cosmological projection } \mathbf{Q}} \longrightarrow \boxed{\text{angular convolution } \mathbf{P}}.$$

Conicus3D is built from AbacusSummit lightcones, so its learned prior $p(\delta)$ also contains a specific structure-formation history. For a test of an alternative gravitational framework, that prior must be distinguished from the measurement operator itself.

Frozen RM input

The current minimal two-derivative local branch is

$$S_g = P \int dT d^3x \sqrt{g} \left[\frac{1}{4Nc_{\text{AE}}^2} \left(\mathcal{K}_{ij} \mathcal{K}^{ij} - \frac{1}{3} \mathcal{K}^2 \right) + N {}^{(3)}R[g] \right], \quad (5)$$

with

$$\mathcal{K}_{ij} = \partial_T g_{ij} - 2D_{(i} B_{j)}, \quad P = \frac{c_{\text{AE}}^4}{16\pi G}. \quad (6)$$

The universal matter interface uses the same cadence field,

$$\mathcal{D}_T \chi^A \equiv \frac{1}{N} \left(\partial_T \chi^A - \mathcal{L}_B \chi^A \right), \quad (7)$$

inside

$$S_m = \int dT d^3x N \sqrt{g} \mathcal{L}_m(\chi^A, \mathcal{D}_T \chi^A, D_i \chi^A; g_{ij}). \quad (8)$$

The electromagnetic principal symbol is

$$\mathfrak{P} = \frac{\omega^2}{N^2 c_{\text{AE}}^2} - g^{ij} k_i k_j = 0. \quad (9)$$

These statements are conditional results of the current RM branch; they are not asserted as established physics.

Local non-regression: what already survives

For the frozen static spherical minimal branch, the isotropic representative can be written

$$g_{ij} = \Omega^4 \delta_{ij}, \quad \Omega = 1 + u, \quad N = \frac{1 - u}{1 + u}, \quad (10)$$

so that

$$n_{\text{min}} = \frac{\Omega^2}{N} = \frac{(1 + u)^3}{1 - u}. \quad (11)$$

Within this branch the static optical solution is the Schwarzschild isotropic form, and its weak expansion gives the standard scalar light-deflection response. Consequently the first-order local weak-lensing operator is a non-regression result, not a discriminator.

This is important because a comparison with Eq. (1) must not manufacture a difference where the local theory predicts none. Any distinct RM signal has to enter through the cosmological lift, the perturbative cadence sector, or the growth history.

Exact removal of the homogeneous cadence factor

Write

$$N(T, \mathbf{x}) = N_0(T) \hat{N}(T, \mathbf{x}), \quad (12)$$

and define

$$\boxed{d\bar{T} = N_0(T) dT}, \quad \boxed{\bar{N} = \frac{N}{N_0} = \hat{N}}, \quad \boxed{\bar{B}_i = \frac{B_i}{N_0}}. \quad (13)$$

Because N_0 is spatially homogeneous,

$$\mathcal{K}_{ij} = N_0 \bar{\mathcal{K}}_{ij}. \quad (14)$$

The two gravitational pieces transform exactly as

$$dT \frac{\mathcal{K}_{ij} \mathcal{K}^{ij} - \mathcal{K}^2/3}{N} = d\bar{T} \frac{\bar{\mathcal{K}}_{ij} \bar{\mathcal{K}}^{ij} - \bar{\mathcal{K}}^2/3}{\bar{N}}, \quad (15)$$

$$dT N {}^{(3)}R = d\bar{T} \bar{N} {}^{(3)}R. \quad (16)$$

The matter interface is equally invariant:

$$\frac{1}{N}(\partial_T - \mathcal{L}_B) = \frac{1}{\bar{N}}(\partial_{\bar{T}} - \mathcal{L}_{\bar{B}}), \quad N dT = \bar{N} d\bar{T}. \quad (17)$$

Thus the frozen local gravity-plus-matter action preserves its form exactly.

Local result. $N_0(T)$ is a redundant homogeneous factor in the frozen CNL-2D + universal-matter branch. One may use $\bar{N}_0 = 1$ without fitting. The status of N_0 in a future global production action remains open.

This removes one apparent cosmological function but does *not* remove the physical inhomogeneous perturbation. With

$$N = N_0(1 + \nu), \quad (18)$$

all nontrivial local lapse/cadence information is carried by $\nu(T, \mathbf{x})$.

Photon frequency, redshift and comoving distance

On a homogeneous background $g_{ij} = a^2(T)\delta_{ij}$ and $N = N_0(T)$, Eq. (9) gives

$$\omega_T = N_0 \frac{c_{\text{AE}} k}{a}. \quad (19)$$

The cadence-counted physical frequency is

$$\omega_{\text{cad}} = \frac{d\varphi}{d\tau_{\text{cad}}} = \frac{\omega_T}{N_0} = \frac{c_{\text{AE}} k}{a}, \quad d\tau_{\text{cad}} = N_0 dT. \quad (20)$$

Therefore, for a conserved comoving mode,

$$\boxed{1 + z = \frac{a_{\text{obs}}}{a_{\text{em}}}} \quad (21)$$

within the local Pure-Cadence branch. Likewise,

$$d\chi = \frac{c_{\text{AE}} N_0}{a} dT = \frac{c_{\text{AE}}}{a} d\bar{T}. \quad (22)$$

The homogeneous factor does not generate an independent distance-redshift law by itself.

What Conicus3D can and cannot test yet

The angular operator $\mathbf{P}$ survives under the same scalar/Born assumptions used in ordinary weak lensing: if the effective optical index is scalar, the deflection is a sky gradient and the shear is the trace-free Hessian of a scalar lensing potential. No new RM coefficient is needed for that statement.

The cosmological projection $\mathbf{Q}$ is different. The present RM corpus still lacks the perturbative equation that fixes $\nu = \delta N/N_0$ on a homogeneous background. It also leaves the homogeneous production law $H(T)$ and the associated structure-growth history open. Therefore Eq. (1) cannot yet be promoted to an RM prediction merely by replacing symbols.

A clean discriminator is the relation between the two scalar pieces of the lensing potential. Schemat-

ically,

$$\mu_{\text{RM}}(k, T) = \frac{1}{2}[\mu_{\Omega}(k, T) + \mu_N(k, T)], \quad (23)$$

where the spatial/constraint part μ_{Ω} is partly controlled by the $\lambda = 1/3$ structure, while μ_N requires the missing perturbative lapse equation. A cosmological slip $\mu_N \neq \mu_{\Omega}$ would become a genuine, calculable RM signature. If the completed action instead forces equality, the weak-lensing kernel may remain degenerate with the standard one at this level.

The learned Conicus3D prior $p(\delta)$ should therefore be treated as an external reference prior, not silently adopted as an RM prior before the RM growth law is derived.

Non-repair and falsification status

The present note makes no attempt to fit Conicus3D, COSMOS-Web, H_0 , Ω_m , or any slip parameter. The relevant conditions are deliberately simple:

- If a completed global action introduces a term that physically fixes $N_0(T)$ and breaks Eq. (13), the local redundancy result must be restricted accordingly.
- If the perturbative lapse equation produces an unacceptable cosmological slip, the branch is rejected rather than repaired with a new coefficient.
- If the completed cosmological sector fails to produce a self-consistent $a(T)$, distance-redshift relation and growth history, the Conicus3D projection operator cannot be claimed as an RM observable prediction.
- If the local scalar optical response ceases to be gradient-generated on the intended branch, the standard $\mathbf{P}$ operator no longer follows.

Conclusion

Using the Conicus3D forward model as an external benchmark separates a local success from a cosmological debt. The current RM branch reproduces the local scalar weak-lensing response on its declared static domain and exactly removes the homogeneous cadence factor $N_0(T)$ from the frozen local gravity-plus-matter action. The remaining physical question is not the background normalisation of N , but its inhomogeneous perturbation $\nu(T, \mathbf{x})$ and the global dynamics that determine $a(T)$ and structure growth.

The resulting programme is narrow and falsifiable: derive the cosmological lapse perturbation, derive the background production law, then decide whether the Conicus3D projection operator remains unchanged or becomes a discriminator. Until those steps are completed, the result is a consistency theorem and a calculation target, not an observational validation.

Acknowledgement of computational assistance. The author is solely responsible for the scientific claims and selections. AI systems were used as tools for algebraic checking, adversarial criticism, drafting and document production.

References

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