

Constitutive Exclusion and Homogeneous Bulk Closure in RM

From the Conicus3D consistency test to a derived zero-mode constraint system

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Abstract. The Conicus3D weak-lensing forward model of Zhao, Scognamiglio, Doré and Bouman is retained only as an external benchmark for the cosmological lift of Relativity of Matter (RM); none of its background equations is imported into RM. Version 4 of this note isolated a provisional homogeneous relation, HCR–1, and required that it be derived from a common global action or rejected. Version 5 performs that test. On the homogeneous Fourier mode $k_F = 0$, the rate-determining elliptic operator of the frozen local branch loses rank, but the algebraic Hamiltonian and conformal scalar constraints survive. With the global constitutive reservoir $\varepsilon_z(e)$, the already normative invariant background density u_0 , and no added kinetic trace term, the surviving constraints are

$$\begin{aligned} P^{(3)}R &= \rho_m + u_\gamma + \varepsilon_z + u_0, \\ \rho_m + 2u_\gamma - 2u_0 &= 2\varepsilon_z + 3\varepsilon'_z. \end{aligned}$$

The second relation algebraically fixes the homogeneous deformation e on a unique sub-capacity branch when its existence condition is satisfied. Preserving it in universal time, with cold matter, isotropic radiation and $\dot{u}_0 = 0$, yields the interior rate law

$$H(3B'(e) + 3\rho_m + 8u_\gamma) = s, \quad B(e) = 2\varepsilon_z + 3\varepsilon'_z,$$

where s is the local matter-to-radiation conversion rate per unit volume. The Hamiltonian constraint is then preserved identically; no Friedmann equation, external energy reservoir or fitted dynamical coefficient is introduced. HCR–1 and the Version-4 relaxation attractor are therefore retired rather than repaired. Boundary production is not part of this homogeneous bulk theorem: a net radiative flux and $\text{EM} \rightarrow \text{AE}$ conversion live in an inhomogeneous boundary layer ($k_F \neq 0$) and determine g_∂ separately. Thus Version 5 closes the homogeneous bulk sector while leaving the boundary-production sector, and therefore the complete cosmological lift, as the next derivation problem. No observational validation of RM is claimed.

Version-5 status. The result claimed here is a *derived homogeneous bulk closure*. It is not a completed cosmology. The boundary-production law remains a separate $k_F \neq 0$ problem.

What changed from Version 4

Version 4 ended with a deliberately sharp requirement: derive HCR–1 from the homogeneous action or reject the branch. The new derivation does not reproduce HCR–1. It produces a different, constraint-based structure. Version 5 therefore makes four explicit changes:

1. HCR–1,

$$\frac{H}{H_{\text{AE}}} = \frac{p_{\text{AE}} - \varepsilon_z}{\varepsilon'_z},$$

is retired from the active derivation.

2. The homogeneous mode is written as $k_F = 0$, where k_F is the Fourier wave number. This is *not* the statement of vanishing spatial curvature.

3. The surviving Hamiltonian and conformal constraints are derived directly on the homogeneous mode.
4. The interior rate H is obtained by preserving the conformal constraint, while the production rate g_{∂} is assigned to a distinct inhomogeneous boundary problem.

The Exclusion barrier itself is retained, but its role changes. It no longer creates a relaxation attractor through HCR–1; instead it enters an algebraic equilibrium constraint whose tangent stiffness controls the response to a changing material/radiative load.

Conicus3D remains an external benchmark

The Conicus3D forward model separates the cosmological projection $\mathbf{Q}$ from the angular weak-lensing operator $\mathbf{P}$,

$$e_{\text{obs}} = \mathbf{P}\mathbf{Q}\delta + \varepsilon. \quad (1)$$

The projection $\mathbf{Q}$ contains a definite background and structure-growth history. RM does not import that history. The useful separation remains

$$\boxed{\text{local optical response}} \longrightarrow \boxed{\text{RM cosmological lift}} \longrightarrow \boxed{\text{angular operator}}.$$

Earlier versions of this note established the local non-regression result and the removability of a homogeneous cadence factor from the frozen local gravity-plus-matter action. Version 5 addresses only the previously open homogeneous bulk step. Conicus3D is therefore a consistency benchmark and eventual discriminator, not a dynamical prior.

Frozen RM input and the homogeneous rank change

The current two-derivative local gravitational branch is

$$S_g = P \int dT d^3x \sqrt{g} \left[\frac{1}{4Nc_{\text{AE}}^2} \left(\mathcal{K}_{ij}\mathcal{K}^{ij} - \frac{1}{3}\mathcal{K}^2 \right) + N {}^{(3)}R[g] \right], \quad (2)$$

with

$$\mathcal{K}_{ij} = \partial_T g_{ij} - 2D_{(i}B_{j)}, \quad P = \frac{c_{\text{AE}}^4}{16\pi G}. \quad (3)$$

The same cadence field enters the universal matter interface through

$$\mathcal{D}_T = \frac{1}{N}(\partial_T - \mathcal{L}_B).$$

The frozen canonical scalar constraints may be written

$$\mathcal{H}_0 = \frac{c_{\text{AE}}^2}{P\sqrt{g}}\pi^{ij}\pi_{ij} - P\sqrt{g} {}^{(3)}R + \mathcal{H}_m \approx 0, \quad (4)$$

$$\mathcal{C} = \frac{3}{2}N\frac{c_{\text{AE}}^2}{P\sqrt{g}}\pi^{ij}\pi_{ij} + \frac{P}{2}N\sqrt{g} {}^{(3)}R - 2P\sqrt{g} D^2N + \mathcal{C}_m \approx 0. \quad (5)$$

On the homogeneous isotropic mode,

$$\pi_{\text{TF}}^{ij} = 0, \quad D^2N_0 = 0.$$

Hence the derivative operator that fixes the local cadence loses rank, while the algebraic parts of both scalar constraints remain. The loss of rank therefore removes a rate-determining local equation; it does not erase the zero-mode constraints.

Domain rule. $k_F = 0$ denotes the homogeneous Fourier mode, not spatial flatness. The homogeneous bulk may have ${}^{(3)}R \neq 0$.

Homogeneous constitutive sector

Let the already manifested bulk be described by

$$g_{ij} = a^2 \gamma_{ij}, \quad H = \frac{\dot{a}}{a}, \quad \dot{e} = 3H. \quad (6)$$

A relabelling-safe definition is

$$e \equiv \ln \left(\frac{V_{\text{el}}}{V_{\text{el, ref}}} \right) \quad (7)$$

for a material element at fixed labels; $e = 3 \ln(a/a_{\text{ref}})$ is its homogeneous coordinate expression.

The Version-4 Exclusion reservoir is retained:

$$\boxed{\varepsilon_z(e) = \kappa \frac{e}{e_{\text{cap}} - e}, \quad \kappa > 0, \quad 0 \leq e < e_{\text{cap}}.} \quad (8)$$

Write $\Delta = e_{\text{cap}} - e$. Then

$$\varepsilon'_z = \kappa \frac{e_{\text{cap}}}{\Delta^2}, \quad \varepsilon''_z = 2\kappa \frac{e_{\text{cap}}}{\Delta^3}. \quad (9)$$

The zero-mode constitutive functional is

$$S_z = - \int dT N_0 V \varepsilon_z(e). \quad (10)$$

No $\dot{e}^2$ term is added. The trace mode has no independent kinetic term in the frozen branch, so adding one would introduce precisely the free coefficient that the present audit is intended to avoid.

At fixed material labels, $dV/V = de$, and therefore the associated homogeneous pressure is

$$\boxed{\rho_z = \varepsilon_z, \quad p_z = - \frac{d(V\varepsilon_z)}{dV} = -(\varepsilon_z + \varepsilon'_z).} \quad (11)$$

Thus the reservoir is tensile, $p_z < 0$. This is the sign result that invalidates the compressive-pressure interpretation used provisionally in HCR-1.

Invariant State-Æ background

Pillar II already separates the local relational state from the homogeneous fundamental density u_0 and states that the latter is not locally stretched by an orbit and obeys $\dot{u}_0 = 0$ in the present architecture. Version 5 uses that existing background; it does not introduce a new compensator.

For a homogeneous dilation at fixed u_0 ,

$$p_0 = - \frac{d(Vu_0)}{dV} = -u_0. \quad (12)$$

Hence the active gravitational combination of the invariant background is

$$u_0 + 3p_0 = -2u_0.$$

This is a background tension carried by a positive density, not a negative-energy fluid.

GHC-1: surviving homogeneous Hamiltonian constraint

The homogeneous Hamiltonian may be written, up to the separate boundary sector,

$$H_G = N_0 V \left[\rho_m + u_\gamma + \varepsilon_z(e) + u_0 - P^{(3)} R \right] + \mu \pi_a. \quad (13)$$

The trace momentum obeys $\pi_a \approx 0$ because the isotropic kinetic combination in Eq. (2) vanishes identically. Variation with respect to N_0 gives

$$\boxed{P^{(3)}R = \rho_m + u_\gamma + \varepsilon_z(e) + u_0} \quad (\text{GHC-1})$$

No H^2 term occurs. GHC-1 is a static bulk relation between intrinsic curvature and the homogeneous content.

The term $-P^{(3)}R$ is the existing curvature contribution to the Hamiltonian constraint. Version 5 does *not* reinterpret it as a new material substance or an external reservoir. Its time variation supplies the compensating geometric term required by preservation of the local constraint balance.

GEC-1: conformal equilibrium constraint

Preservation of $\pi_a \approx 0$, equivalently the homogeneous conformal scalar constraint, gives

$$P^{(3)}R + 3p_{\text{tot}} = 0, \quad (14)$$

with

$$p_m = 0, \quad p_\gamma = \frac{1}{3}u_\gamma, \quad p_z = -(\varepsilon_z + \varepsilon'_z), \quad p_0 = -u_0.$$

Combining Eq. (14) with GHC-1 yields

$$\boxed{\rho_m + 2u_\gamma - 2u_0 = 2\varepsilon_z(e) + 3\varepsilon'_z(e)} \quad (\text{GEC-1})$$

Define

$$B(e) \equiv 2\varepsilon_z(e) + 3\varepsilon'_z(e). \quad (15)$$

For Eq. (8),

$$B'(e) = 2\varepsilon'_z(e) + 3\varepsilon''_z(e) = 2\kappa \frac{e_{\text{cap}}}{\Delta^2} + 6\kappa \frac{e_{\text{cap}}}{\Delta^3} > 0. \quad (16)$$

Furthermore

$$B(0) = \frac{3\kappa}{e_{\text{cap}}}, \quad B(e) \rightarrow +\infty \quad (e \rightarrow e_{\text{cap}}^-).$$

Therefore, whenever

$$\boxed{\rho_m + 2u_\gamma - 2u_0 \geq \frac{3\kappa}{e_{\text{cap}}},} \quad (17)$$

there exists one and only one solution $e_{\text{eq}} \in [0, e_{\text{cap}})$ of GEC-1.

LF-09 in Version 5. The homogeneous branch is not stopped at the capacity by an imposed rule. The conformal constraint selects at most one admissible state below the capacity, and the branch ceases to exist if its algebraic existence condition fails.

HBR-1: derived homogeneous bulk rate

The rate is obtained by preserving GEC-1, not by introducing a new expansion equation. Let s be the local matter-to-radiation conversion rate per unit volume. For cold matter and isotropic radiation in the homogeneous bulk,

$$\dot{\rho}_m = -3H\rho_m - s, \quad \dot{u}_\gamma = -4Hu_\gamma + s, \quad \dot{u}_0 = 0, \quad \dot{e} = 3H. \quad (18)$$

Differentiating GEC-1 gives

$$3HB'(e) = -3H\rho_m - 8Hu_\gamma + s,$$

and therefore

$$\boxed{H [3B'(e) + 3\rho_m + 8u_\gamma] = s} \quad (\text{HBR-1})$$

Since $B' > 0$, $\rho_m \geq 0$ and $u_\gamma \geq 0$, the denominator is strictly positive on the regular branch:

$$H = \frac{s}{3B'(e) + 3\rho_m + 8u_\gamma}. \quad (19)$$

Thus

$$s > 0 \Rightarrow H > 0, \quad s = 0 \Rightarrow H = 0,$$

within the assumptions of the homogeneous bulk ledger.

This result replaces the Version-4 relaxation law. There is no independent relaxation time for e at this order: e is algebraically constrained by GEC–1, and its time dependence follows from the time dependence of the load.

Preservation of GHC-1

For the conformal bulk flow, $\dot{R} = -2HR$. Differentiating the right-hand side of GHC–1 using Eq. (18) gives

$$\frac{d}{dT}(\rho_m + u_\gamma + \varepsilon_z + u_0) = -3H\rho_m - 4Hu_\gamma + 3H\varepsilon'_z \quad (20)$$

$$= -2H(\rho_m + u_\gamma + \varepsilon_z + u_0), \quad (21)$$

where the second line is exactly GEC–1. Hence

$$P\dot{R} = -2HPR$$

and GHC–1 is preserved identically. The two scalar constraints are therefore mutually consistent; no additional homogeneous rate equation appears.

The corrected energy reading

A previous audit temporarily treated radiative transfer from the bulk to the boundary as if it were an export from the complete manifested domain. That is incorrect. The physical control volume matters.

For an old homogeneous bulk element at fixed labels, the curvature term in GHC–1 supplies the geometric work required by preservation of the constraint. The relevant statement is not that energy leaves the Universe, but that exchanges between content and the existing relational-curvature term are internal to the manifested system.

Accordingly, the propagation ledger is read as a sequence of internal work exchanges. In the homogeneous bulk, the redshift-related term P_z is the photonic work ceded to the continuum, while the tensile work $3HV(\varepsilon_z + \varepsilon'_z)$ is transferred from the continuum to the constitutive reservoir. Their difference,

$$W_z \equiv P_z - 3HV(\varepsilon_z + \varepsilon'_z), \quad (22)$$

is therefore a continuum balance term, not an externally injected source and not, in the zero-mode closure, an independently prescribed return flux to the photons.

This reading is not required as an additional equation for HBR–1; it is the bookkeeping that makes the historical two-channel ledger compatible with the already derived Hamiltonian and conformal constraints.

Boundary production is a separate $k_F \neq 0$ sector

The total manifested-volume ledger remains

$$H_{\text{AE}} = H + g_\partial, \quad g_\partial = \frac{\Gamma_\partial}{3V_{\text{AE}}}. \quad (23)$$

Version 5 does not set $g_\partial = 0$. It instead separates the two physical problems.

In a homogeneous isotropic bulk, the net radiative flux vector vanishes by isotropy. A non-zero outward photon flux and the $\text{EM} \rightarrow \text{AE}$ conversion therefore live in a boundary layer with gradients, hence in an inhomogeneous sector. The bulk equations GEC–1, GEC–1 and HBR–1 describe the deep homogeneous region; the boundary layer determines g_∂ through its own transport and conversion laws.

The current boundary ledger is retained as a conditional target,

$$\mathcal{U}_\partial \Gamma_\partial = \mathcal{P}_{\gamma, \partial}, \quad \mathcal{U}_\partial = \frac{P}{\ell_\star^2}, \quad (24)$$

but Version 5 does not identify the formation cost $\mathcal{U}_\partial$ with the residual energy density u_{birth} of the newly manifested volume. That distinction belongs to the boundary derivation.

For a growing domain, the volume-averaged curvature need not obey the fixed-label law. If newly produced volume has curvature R_N , then for $\bar{R} = V^{-1} \int R dV$ the production contribution is

$$\boxed{\Xi_\partial = 3g_\partial(R_N - \bar{R})}, \quad (25)$$

under the homogeneous-shell bookkeeping used here. This affects the global average, not the pointwise bulk constraint. It is another reason not to insert boundary production into the $k_F = 0$ bulk equation by hand.

Supersession of HCR-1 and the Version-4 attractor

Version 4 obtained a stable sub-capacity attractor only conditionally on HCR–1. Version 5 now performs the action-level test required by that version and obtains a different structure. The correspondence is

Version 4	Version 5
HCR–1 postulated	HCR–1 retired
$p_{\text{AE}} > 0$ compressive load	derived tensile reservoir pressure $p_z = -(\varepsilon_z + \varepsilon'_z)$
sub-capacity equilibrium from $p_{\text{AE}} = \varepsilon_z$	sub-capacity state from GEC–1
relaxation time $\sim (3H_{\text{AE}})^{-1}$	no independent zero-mode relaxation time
$r = H/H_{\text{AE}}$ from HCR–1	H from HBR–1; g_∂ from boundary sector
capacity dynamically repulsive	capacity excluded by algebraic existence on the regular branch

The Version-4 work was therefore not a failed calculation: it isolated the exact object that had to be derived. The derivation replaces that object instead of adjusting it.

What the homogeneous closure does and does not claim

The Version-5 bulk system is

$$P^{(3)}R = \rho_m + u_\gamma + \varepsilon_z(e) + u_0, \quad (1)$$

$$\rho_m + 2u_\gamma - 2u_0 = 2\varepsilon_z(e) + 3\varepsilon'_z(e), \quad (2)$$

$$H [3B'(e) + 3\rho_m + 8u_\gamma] = s, \quad (3)$$

$$\dot{e} = 3H, \quad (4)$$

$$H_{\text{AE}} = H + g_\partial. \quad (5)$$

Equations (1)–(4) close the homogeneous bulk once the local source s and the constitutive parameters of ε_z are specified. Equation (5) is the exact volume decomposition; g_∂ is not supplied by the bulk and must be obtained from the boundary sector.

No Friedmann equation, H_0 , Ω_m , external equation of state or fitted response coefficient is used to obtain Eqs. (1)–(4).

Remaining debts and falsification conditions

The debts after Version 5 are narrower than those of Version 4:

1. **Boundary layer.** Derive the $k_F \neq 0$ conversion layer, its radiative transport Φ_∂ , its curvature gradient, and the resulting g_∂ .
2. **Formation cost versus birth density.** Determine whether $\mathcal{U}_\partial$ is a transient conversion cost, a residual density, or partly routed to another state; do not set $\mathcal{U}_\partial = u_{\text{birth}}$ without derivation.
3. **Constitutive scale.** Derive or independently anchor κ and e_{cap} ; they may not be fitted solely to reproduce an external expansion history.
4. **Propagation ledger.** Rewrite the historical P_z/W_z notation so that the continuum-work interpretation used by the preserved constraints is explicit.
5. **Perturbations and lensing lift.** Derive the $k_F \neq 0$ cosmological perturbation equations and the RM distance/redshift and structure-growth operators before returning to Conicus3D as a discriminator.

The homogeneous bulk branch is falsified or replaced if any of the following occurs:

1. a complete derivation of the frozen action shows that either GHC–1 or GEC–1 has an omitted homogeneous term;
2. the invariant background u_0 does not enter the homogeneous interface with $p_0 = -u_0$ under the declared Pillar-II architecture;
3. the physical load violates Eq. (17), so no admissible $e < e_{\text{cap}}$ exists;
4. the boundary theory requires a modification of the bulk constraints rather than a separate inhomogeneous matching problem;
5. an additional free dynamical coefficient is required only to reproduce a chosen external cosmological curve.

Conicus3D after Version 5

The local optical response remains a non-regression result, and the homogeneous bulk is no longer represented by a provisional HCR–1. The remaining route to an observational comparison is now

$$\begin{aligned} \text{homogeneous bulk closure} &\longrightarrow \text{boundary production} \longrightarrow H_{\text{AE}}(T) \\ &\longrightarrow \text{perturbation/growth lift} \longrightarrow \text{Conicus3D benchmark.} \end{aligned}$$

Conicus3D becomes a genuine discriminator only after the boundary and perturbative sectors are derived internally. If a standard distance-redshift relation is recovered later, it must emerge after the RM derivation rather than being imposed as a selection criterion.

Conclusion

Version 4 deliberately placed the homogeneous branch under a kill condition: derive HCR–1 or reject it. Version 5 rejects HCR–1 and retains the branch through a different result.

On the homogeneous Fourier mode, the derivative cadence operator loses rank but the algebraic Hamiltonian and conformal scalar constraints survive. With the positive constitutive reservoir $\varepsilon_z(e)$ and the invariant background u_0 , they give GHC–1 and GEC–1. For the Exclusion barrier, GEC–1 has at most one admissible sub-capacity solution and supplies an explicit existence threshold. Preserving that constraint yields the parameter-free bulk rate HBR–1,

$$H [3B'(e) + 3\rho_m + 8u_\gamma] = s,$$

while GHC–1 is then preserved identically.

The homogeneous bulk therefore closes without an imported Friedmann equation, without an external energy reservoir and without a new fitted dynamical coefficient. Boundary production is neither set to zero nor derived by this theorem. It belongs to the inhomogeneous conversion layer and remains the next calculation needed to determine the total manifested-volume rate $H_{AE} = H + g_\partial$.

This is the precise Version-5 claim: the homogeneous Hamiltonian debt exposed by the Conicus3D exercise is closed at bulk level; the boundary-production debt is isolated rather than hidden.

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