

Fundamental Energetic Symmetry

Æ — The Missing State of the Universe

How State Æ Pays for Gravity

From the Relational Tensor Q_{ij} to Newton's Potential
and the Universal Energy Ledger

RM — The Relativity of Matter, Pillar II / Book II

Version v4.7.0 — Bulk Concordance Edition

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DOI: 10.5281/zenodo.21263352

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September 23, 2026

Preface — The Return to the Sphere

I am a *Homo sapiens* of the year 2026.

Twenty thousand years ago, I imagine one of my distant ancestors looking up at the Moon. He probably did not see merely a silver disc fixed to the vault of the sky. The movement of light, the slow advance of shadow, and that presence suspended in the night may already have revealed depth to him. Behind the appearance of the disc, his physical intuition sensed a sphere: another material world, tangible and perhaps similar to his own.

He had neither telescope, nor equation, nor theory of gravitation. Yet he possessed the instrument that precedes every discovery: the ability to imagine that reality is deeper than the image it presents to us.

Every age nevertheless runs the same risk: mistaking its representation of reality for reality itself. Mathematics is one of the most extraordinary achievements of human intelligence. It describes, orders, predicts, and reveals relationships that our senses alone could never reach. But a mathematical representation, however elegant, does not necessarily become a physical object. A geometry may describe the behaviour of reality without constituting its substance. An equation may permit a solution without nature being required to realise it.

Contemporary physics has sometimes granted its geometrical constructions an ontological sovereignty that they themselves may never have claimed. Matter, space, and time then risk disappearing behind their representation. What an equation permits becomes imaginable; what becomes imaginable eventually comes to be described as physically possible.

“Aladdin’s Tunnel”, which I shall dismantle later with deliberate mathematical irony, embodies this confusion. Wormholes may constitute fascinating solutions within certain geometrical extensions. But when topology must be cut, folded, and stitched back together to manufacture a passage between two regions of the Universe, one elementary question remains: have we discovered a physical object, or merely explored how far our mathematical language can go?

This book was born from that question.

It gathers an architecture of ideas, intuitions, postulates, equations, acknowledged contradictions and calculable paths. Some avenues will survive; others will have to be abandoned. I ask that they be examined for what they reveal, rather than rejected because they look at reality from a different starting point.

In RM, time is an open line. Clocks do not measure a substance called time: they count the rate of physical phenomena. Matter becomes relative to the state of the medium in which it exists. State $\mathcal{A}$ is postulated as the physical continuum through which RM seeks a material origin for manifestations that geometry describes admirably, but may not explain entirely.

This work is therefore a return to the sphere.

An invitation to look behind the apparent disc; not to confuse the map with the territory, the

measurement with the object measured, or the power of an abstraction with the physical existence of what it represents.

I do not ask the reader to believe.

I ask the reader to look a second time.

The Relativity of Simplicity

Nothing is more complex than making things simple.

I worked independently, with limited mathematical means, to develop an idea that has lived within me since childhood: What becomes of light at the boundaries of our Universe? If it is not reflected upon contact with Silentium, what can it become, if not State $\mathcal{A}$?

My method rested on three pillars: the principle of parsimony, primal common sense, and the courage of the absurd. The simplest minds, too, sometimes possess a little something of Mozart. My hope is that a genuine physicist may discover that little something within RM.

I wish to end with the hope that has accompanied me throughout my life: work always pays in the end, as Hesiod wrote. The accumulation of knowledge matters; the decisive act remains reasoning about what that knowledge means.

Time is only a date.

Pierre Benaros

What This Document Establishes, What It Postulates, and What Would Refute It

The preface says why this work exists. This page says what it is worth. A reader is entitled to know, before the first equation, which statements are derived, which are posed, and which observation would end the matter. Nothing below is rhetorical: every item is checkable against a named section.

BOOK II SIGNATURE — THE REVISED SCHWARZSCHILD RELATION

Throughout Book II, RM uses the Schwarzschild observable factor as a cadence relation rather than as a statement that time itself slows down. For the isolated-source exterior branch,

$$C_{\text{RM}}(r) = \sqrt{1 - \frac{2GM}{rc^2}} = \sqrt{1 - \frac{R_S}{r}} = \sqrt{1 - C(r)}.$$

The editorial slogan is deliberately simple: **Time does not slow down. Matter changes cadence.** At v4.7.0 this remains a named RM reinterpretation and a positive provocation, not a claim that its full strong-field derivation has already been completed. The derivation, domain of validity and independent observational tests remain part of the calculation frontier.

I. What is derived

These results follow from the stated action by calculation, not by assumption.

- **The Poisson limit follows from the local quadratic candidate action CLG-1.** For non-homogeneous modes $k \neq 0$, the spatially gauge-invariant scalar $\mathcal{G}[q] = \nabla^2 \text{Tr } q - \partial_i \partial_j q_{ij}$ annihilates the homogeneous mode without an imposed projector. With $\Psi_{\mathcal{E}} = \sigma \Lambda$ and the unique matching $Z\sigma^2 = 8\pi G$, it gives $\nabla^2 \Psi_{\mathcal{E}} = 4\pi G \rho_m$.
- **The attractive pair energy $E_{12} = -Gm_1 m_2 / r$ follows with the same normalisation.** No second gravitational constant or sign choice is introduced.
- **The local Dirac matrix has rank ten.** For $k \neq 0$, $B \neq 0$ and $\mu_A \neq 0$, six first-class and ten second-class constraints leave exactly two local propagating degrees of freedom. The two tensor polarisations have positive quadratic energy for $Z > 0$ and front speed $c_{\mathcal{E}}$.
- **The minimal nonlinear curvature candidate does not alter the quadratic closure.** For the relational state tensor $\gamma_{ij} = \delta_{ij} + q_{ij} + O(q^2)$, its intrinsic scalar satisfies $\mathcal{R}^{(1)}[\gamma] = -\mathcal{G}[q]$. The dimensionless invariant $X_{\mathcal{R}} = \ell_{\star}^4 \mathcal{R}_{ij} \mathcal{R}^{ij}$ is therefore $O(q^2)$, while the subtracted Born-Infeld function $1 - \sqrt{1 - X_{\mathcal{R}}} - X_{\mathcal{R}}/2$ begins at $X_{\mathcal{R}}^2/8 = O(q^4)$. Hence it introduces

no quadratic mass, tadpole, kinetic renormalisation or change to the CLG-1 Dirac matrix about the homogeneous branch.

- **The static spherical N-EMPR reduction reaches the constitutive regime without adding a stellar fitting coefficient.** With the frozen toy EOS $p = \rho^2$, the same constitutive function $F(X) = 1 - \sqrt{1 - X} - X/2$ and frozen reduced coefficients, v28 finds two converged hydrostatic solutions at $p_c = 0.5028$ on opposite sides of $Q_c = 0$, then continues the high branch to $X_c = 0.9715$ with $f_{\min} > 0$. These are reduced-unit conditional numerical results, not kilometre or solar-mass predictions, and no external human reproduction is claimed in this edition.
- **The regular $Q_c < 0$ branch is cross-method reproducible at the declared test pressure.** At $p_c = 0.5028$, a post-v28 DOP853 shooting implementation gives $R = 2.08849775$ and $M = 0.84685706$, while a separately implemented two-domain collocation gives $X_c = 0.9109684074$, $Q_c = -0.0598152950$, $R = 2.0884976108$ and $M = 0.8468571424$, with $\max |H| < 8 \times 10^{-14}$ in both domains. Both reproduce a smooth internal $Q = 0$ crossing. This confirms branch existence and regularity at the stated test point; it does not rewrite the dated v28 coordinates, establish the full high-branch continuation, radial stability, a physical EOS calibration or an astrophysical mass–radius prediction.
- **Photonic cadence becomes a conditional first-passage theorem on a frozen background.** If local material transport is represented by the Fick generator $\mathcal{L}_{\text{loc}} f = \nabla_i (D_{\text{loc}} \nabla^i f)$ and local process cadence satisfies $d\tau_{\text{cad}} = N dT$, Dynkin’s equation for the mean universal escape time is $\mathcal{L}_{\text{loc}} \Theta = -1/N$. Holding g_{ij} and D_{loc} fixed, the ratio $\Theta_N / \Theta_{N=1}$ is exactly a positive kernel-weighted mean of $1/N$. No optical fitting coefficient is introduced. This is a transport theorem under its stated assumptions, not a complete stellar diffusion calculation or an experimental validation of RM.
- **The Black-Core candidate carries two distinct operators and two distinct scales.** The compactness threshold $R_S \propto M$ is an optical and transport boundary. The curvature barrier $X_{\mathcal{R}}(r_c) = 1$ is the mechanical phase boundary and gives $r_c \propto M^{1/3}$. Consequently M/r_c^3 is independent of M : within the stated spherical reduction, the E -rich core has the scaling signature of a three-dimensional incompressible volume. If a residual fraction $M_{\text{res}} = \varepsilon_M M$ occupies that core, its residual density is also independent of M . Under the isotropic saturated-core constraint, matching that density to $X_{\mathcal{R}} \rightarrow 1^-$ gives the conditional identity $\varepsilon_M = 2\sqrt{\pi G Z} / c_{\text{E}}^2$ and the admissibility bound $Z \leq c_{\text{E}}^4 / (4\pi G)$. On the independently normalized branch $|\sigma| = 16\pi G / c_{\text{E}}^2$, the fraction becomes $\varepsilon_M = \sqrt{2}/4$ without a new Black-Core parameter.
- **The Born–Infeld energy and scalar bulk projection admit exact spherical integrals.** For $X_{\mathcal{R}}(r) = (r_c/r)^6$ outside the mechanical boundary,

$$\int_1^\infty y^2 F(y^{-6}) dy = \frac{\pi - 3}{6}, \quad \int_1^\infty y^2 [4XF'(X) - 3F(X)]_{X=y^{-6}} dy = \frac{5 - \pi}{6}.$$

Thus both bulk contributions scale as $u_\star r_c^3 \propto M$. Their normalized coefficients, $(\pi - 3)/2$

and $(5 - \pi)/2$, sum exactly to one. This is a derived bulk identity; it is not yet the complete OCH-1 junction because the metric variation also generates a boundary flux at r_c .

- **Local mechanical conservation closes without changing the fundamental density.** The demonstrated identity is $d(K + E_{12}) = 0$. The homogeneous density u_0 is not locally depleted by a fall.
- **The constrained scalar already determines the relational state inside a non-saturated star.** In the isotropic representative $q_{ij} = \psi \delta_{ij}$, one has $\mathcal{G}[q] = 2\nabla^2 \psi$ and therefore $\psi = \Phi[q]$ for $k \neq 0$. On the independently normalised branch $|\sigma| = 16\pi G/c_{\mathcal{E}}^2$, this gives $\psi = -\text{sgn}(\sigma)C$. No stellar saturation postulate is required.
- **The principal eikonal symbol of the electromagnetic operator is fixed without a free optical coefficient.** Relabelling covariance forces the electromagnetic sector to couple as $-\frac{1}{2}q_{ij}\Sigma^{ij}$ with the Maxwell stress tensor and with coefficient exactly one; the Exclusion Interaction (historical PS-0 terminology: Principle of Saturation) fixes the cadence powers. The symbol is $\mathfrak{P} = \omega^2/(N^2 c_{\mathcal{E}}^2) - g^{ij}[q]k_i k_j$ with $N^2 = 1 - C$ and $g^{ij} = \delta^{ij} - q^{ij} + O(q^2)$. It is the optical structure of a medium, obtained on $\Omega_{\mathcal{E}}(T)$ with absolute time and no fourth physical dimension.
- **γ_{PPN} is quantised in RM.** The symbol gives $n^2 = (1 + \psi)/(1 - C)$ and therefore $\gamma_{\text{PPN}} = \psi/C = -\text{sgn}(\sigma)$. Since $|\psi| = |C|$ on the weak branch, RM admits no continuum of values: $\gamma_{\text{PPN}} \in \{-1, +1\}$ and cannot be tuned. On $\sigma < 0$ the prediction is $\gamma_{\text{PPN}} = +1$ exactly, that is 0.91σ from the Cassini central value; on $\sigma > 0$ the index is identically unity and the solar deflection vanishes. See N- $\mathcal{E}$ -02, and the conditionality recorded in debt D- $\mathcal{E}$ -02.
- **The same sourced gauge branch conditionally closes the preferred-frame pair.** Gauge invariance of the first-order matter coupling, together with the three-dimensional material momentum balance, requires equal shift and stress couplings. Universal minimal coupling fixes their common value and gives $b_i = \frac{7}{2}V_i + \frac{1}{2}W_i$, hence $\alpha_1 = \alpha_2 = 0$. This result is not a fit; it remains conditional on the complete microscopic matter action.
- **The areal-isotropic radius map gives $\beta_{\text{PPN}} = 1$.** The exact cadence law is written with the physical areal radius, whereas the PPN series uses the isotropic radius. Their explicit conversion gives $C_A = C_\rho - \frac{1}{2}C_\rho^2 + O(C_\rho^3)$ and therefore $N^2 = 1 - C_\rho + \frac{1}{2}C_\rho^2 + \dots$. The same value follows from the minimal constant-coefficient CNL-2D candidate. It is a conditional theorem, not a fitted recovery of Mercury.
- **The trace-free kinetic term has a conformal primary constraint, and the v4.2.0 nonlinear local count remains two.** In the minimal two-derivative candidate, $\lambda = 1/3$ makes $\pi = g_{ij}\pi^{ij} \approx 0$ primary. The associated elliptic block is invertible only for $k \neq 0$; its homogeneous kernel permits $\mathcal{K} = 3H(T)$ without fixing $H(T)$ locally. N- $\mathcal{E}$ -06 computes the local nonlinear scalar block and the minimal constrained antisymmetric completion; on the regular elliptic branch the total count is six first-class plus ten second-class constraints,

hence $N_{\text{grav}} = 2$. This remains conditional on D-Æ-04M and on the absence of a kernel in the relevant elliptic blocks.

- **The same matching reproduces the Newtonian field-energy and stress ledgers.** CLG-1 gives $u_{\text{rel}} = -|\nabla\Psi_{\text{Æ}}|^2/(8\pi G)$ and the inverse-Maxwell stress $\Pi_{ij} = -(4\pi G)^{-1}[g_i g_j - \delta_{ij} g^2/2]$. Hence $\partial_j(P\delta_{ij} - \Pi_{ij}) = 0$: hydrostatic balance is the local matter-Æ stress balance. In spherical symmetry the relational sector is radially compressive and tangentially tensile.
- **A main-sequence star is non-saturated on the retained Planck-scale branch.** With standard solar inputs and $\ell_{\star} = \ell_P$, the central value is $X_{\mathcal{R}} \simeq 7.1 \times 10^{-181}$. This excludes the former stellar phase-rupture mechanism on that branch; it does not constitute a new stellar model.
- **The homogeneous bulk rate is derived on the reduced $U = 0$ branch.** With $B(e) = 2\varepsilon_z + 3\varepsilon'_z$, preservation of the surviving homogeneous constraint under the declared matter-radiation ledger gives $H[3B'(e) + 3\rho_m + 8u_\gamma] = s$. This is HBR-1. The thermal/curvature completion extends this equation with $\varepsilon_{\text{Æ}}^{\text{th}}$ and $U(X)$ and reduces exactly to HBR-1 when $U = 0$.
- **The local P-CIN kinetic radical passes its canonical health audit.** In the isolated trace-free kinetic sector the velocity-momentum map is globally invertible for finite momentum, the Hamiltonian is positive and convex, the kinetic Hessian is positive for $y < 1$, and the lapse remains non-kinetic. This establishes a healthy local deviatoric foundation for the strong-field programme; it does not by itself prove preservation of the complete field constraint chain or the two-polarisation count on the full P-CIN theory.
- **A historical radiative-constitutive effective model was confronted with data.** Its Pantheon+ checkpoint gave $w_{\text{Æ,eff}} = -0.683953$, $z_t = 0.834140$ and $\Delta\chi^2 \simeq +0.244$ relative to the comparison model. v4.7.0 retains this only as a viability record of a superseded effective branch; it is not the active homogeneous closure and does not calibrate HBR-1 or the thermal completion.
- **Standard black-hole thermodynamics is recovered from the RM identification** $C = -2\Psi_{\text{Æ}}/c^2$. Given that identification, $|\nabla C(R_S)| = 2\kappa/c^2$ and the Bekenstein-Hawking entropy follows. This is a consistency result, verified numerically to machine precision. It is not a resolution of the unitarity problem, and Chapter 9 says so explicitly.

II. What is postulated, and openly so

RM is incomplete, and the incompleteness is not observational. It is constitutive: the complete action of the medium and its state transitions are not yet fixed. The following items carry that weight.

- **CLG-1 is a local quadratic closure, not the complete nonlinear action.** Its gauge structure, trace-free kinetic term, scalar multiplier and constrained antisymmetric sector

are explicit constitutive ingredients. CNL-BI1 supplies a minimal gauge-invariant nonlinear candidate, not a completed theory. The N-EMPR spherical sector now provides conditional static strong-field solutions with the same constitutive barrier, while the generic nonlinear constraint algebra, physical-EOS calibration and independent strong-field reproduction remain open.

- **CNL-2D survives N-Æ-06 conditionally.** Spatial relabelling still permits the general functions $A(N), B(N), C(N), D(N), V(N)$. N-Æ-05 adds the Pure Cadence Principle (PCP): N is a universal rate converter, not an autonomous local energy reservoir. Under PCP and the cadence translation already registered in N-Æ-02, the functions collapse to $A = P/(4Nc_{\mathcal{A}}^2)$, $B = P/(12Nc_{\mathcal{A}}^2)$, $C = PN$, $D = V = 0$. N-Æ-06 then tests this frozen branch: on the regular local $k \neq 0$ elliptic branch, the nonlinear gravitational constraint chain retains six first-class and ten second-class constraints and therefore two local gravitational degrees of freedom. D-Æ-04A is closed *conditionally*; D-Æ-04M remains open and narrowed to explicit microscopic matter sectors that may modify the canonical structure.
- **The electronic closure PF-e-R1 rests on eight clauses**, one of which — the support of the $\mathcal{A}$ response — remains undecided among four issues. The frozen value $R_{NC}^{\text{anc}}(A) = 262211.6$ ly is reproducible to the last digit and physically conditional to the same degree.
- **The normalisation of $c_{\mathcal{A}}$ is not broken.** The anchoring density scales as $u_{\star} \propto c_{\mathcal{A}}^5$; the construction changes the prefactor and nothing else. No branch of this work supplies an independent determination.
- **The former local potential $\mathcal{F}_{\text{sat}}(Q)$ is not restored.** Its dependence on raw $\text{Tr } Q$ and s_{ij} breaks the spatial labelling gauge; its expansion about $Q = 0$ also contains a tadpole. CNL-BI1 instead builds a candidate barrier exclusively from relational curvature invariants. The replacement is structural, not a relabelling of the retired potential.
- **P-CIN is a strong-field kinetic candidate, not a consequence of the static $U(X)$ branch.** Its local trace-free kinetic radical passes the canonical health audit, but the full separated-point constraint algebra, trace/curvature/matter coupling and hyperbolicity remain open. The pointwise audit therefore does not independently establish the complete two-polarisation theorem.
- **UC-1 and the thermal-to-state bridge remain postulated.** The capacity identification $u_{\mathcal{A}}^{\text{cap}} = P/\ell_{\star}^2$ is an RM constitutive postulate. The Maria bound is conditional on it, and the further sequence from approach to that bound through LF-09 to $EM \rightarrow \mathcal{A}$ remains a working bridge until a microscopic conversion law is derived. The unit boundary identification $\mathcal{U}_{\partial} = P/\ell_{\star}^2$ is likewise a conditional branch, not a universal theorem of formation cost.
- **The double-scale Black-Core closure is conditional.** It assumes an isotropic regular core, a universal residual fraction and the same scalar coupling σ as CLG-1. Zone II is no longer interpreted as a progressive $\mathcal{A} \rightarrow E$ conversion layer: State $\mathcal{A}$ remains present there

and carries a growing nonlinear relational stress. The actual mechanical conversion occurs at r_c , where $X_{\mathcal{R}} = 1$. The Born–Infeld boundary flux and the opposing wall tension have not yet been matched from a unique phase action.

- **OCH-1 separates exterior mass from local scalar charge.** The complete macroscopic mass M governs the exterior relational tide and therefore the scale $r_c \propto M^{1/3}$. Inside the saturated core, only the unconverted material residue $M_{\text{res}} = \varepsilon_M M$ sources the scalar constraint $-\mathcal{R}_c = \sigma \rho_{\text{res}}$. The cohesive and conversion sectors remain in the total energy and radial-stress ledger, but carry no independent CLG-1 scalar charge on this branch. This is a constitutive postulate, not a derived equivalence principle.

III. What would refute it

A theory that cannot name its own death is not a theory. These are named.

- **Failure of the homogeneous bulk closure.** HBR-1 fails if the surviving zero-mode constraints cannot be preserved under the declared matter–radiation ledger, or if the derivation requires an imported Friedmann equation, H_0 fit or hidden homogeneous response coefficient. The thermal extension fails if it does not reduce to HBR-1 at $U = 0$ under the same bookkeeping.
- **Failure of P–CIN beyond the local audit.** The present local result does not protect the full strong-field theory. If the complete field constraint chain changes rank or class, a ghost or negative branch appears, the lapse becomes dynamical, or the evolution equations fail the required hyperbolicity test, the P–CIN completion must be modified or withdrawn rather than rescued.
- **Any chromatic signature.** A photon mass, a frequency-dependent speed, cosmological dispersion, or vacuum birefringence terminates the saturated photon branch.
- **The speed of tensor modes.** CLG-1 gives the characteristic relation $\omega^2 = c_{\mathcal{A}}^2 k^2$. Once $c_{\mathcal{A}}$ is normalised independently, gravitational-wave timing directly tests this equality.
- **Tensor dispersion or mass.** CLG-1 is massless at quadratic order. A measured non-luminal dispersion, or a nonlinear completion that introduces an unprotected tensor mass, would replace this branch rather than tune it.
- **A failure of the constraint algebra.** If the complete bracket matrix ceases to have six first-class and ten second-class constraints, or if $B_{\mu A} = 0$ changes the branch without a new consistent Dirac analysis, the two-polarisation claim of CLG-1 fails.
- **Extra local polarisations.** Any independently propagating scalar or antisymmetric mode in the completed theory would contradict the present CLG-1 branch and would be directly testable.

- **Failure at the Cassini frontier.** RM has derived $g_{00}^{\text{eff}} = 1 - C$ but not the spatial part of the optical propagation operator. The naive identification $\gamma_{ij} = \delta_{ij} + q_{ij}$ gives the wrong weak-field spatial sign and is therefore forbidden. If the eikonal reduction of the completed operator cannot produce the observed coefficient $\gamma_{\text{PPN}} = 1 + (2.1 \pm 2.3) \times 10^{-5}$ without a fitted optical parameter, the present local branch fails.
- **Failure of the conditional PPN closure (v4.0.0 successor to the preceding historical items).** The local branch fails if the optical coupling or index of N-Æ-02 fails on recalculation; if $\sigma < 0$ is independently excluded; if the universal microscopic matter action changes the sourced momentum constraint or gives $\alpha_1 \neq 0$ or $\alpha_2 \neq 0$; if the areal-isotropic conversion fails to give $\beta_{\text{PPN}} = 1$; or if an independently required cadence term changes that value. No second sign, vector normalisation or second-order coefficient may be introduced to repair the branch.
- **Failure of the minimal two-derivative completion.** CNL-2D is rejected if its complete nonlinear Dirac algorithm changes the constraint rank, produces an inconsistent tertiary chain or releases an additional local scalar; if the microscopic matter action does not reproduce the frozen source coupling; or if the cosmological boundary sector cannot supply $H(T)$ on the homogeneous kernel. The action may not be declared unique merely because its static solution agrees with observation.
- **Failure of nonlinear protection.** CNL-BI1 is rejected if its full Dirac analysis changes the intended local degree count, if the barrier is crossed dynamically, if its spherical branch fails to regularise the central response, or if it introduces a tensor mass or a second characteristic cone about the physical background.
- **Failure of the cross-method stellar branch-existence result.** At $p_c = 0.5028$, DOP853 shooting and two-domain collocation now reproduce the same regular $Q_c < 0$ solution within the stated numerical tolerances. The result fails if a certified independent implementation of the same undivided equations, EOS and frozen coefficients cannot reproduce that branch or reveals a genuine discontinuity, rank loss or unresolved boundary mismatch at the declared test point. The dated v28 global coordinates remain in the audit record and are not retroactively altered.
- **Failure of the double-scale junction.** The Black-Core branch is rejected if the derived Born-Infeld boundary flux and radial pressure balance fail to select the same $r_c \propto M^{1/3}$, require a mass-dependent residual fraction, or need a freely adjusted surface tension to reproduce the mechanical boundary.
- **Failure of ontological source separation.** OCH-1 is rejected if the variational completion forces the cohesive state E to enter the local scalar constraint with the same charge as residual matter, if the exterior solution responds only to $\varepsilon_M M$ rather than to M , or if the complete radial ledger cannot conserve energy and stress across the nonlinear Zone II and the mechanical surface at r_c without an additional fitted coupling.

- **The five conditions of PF-e-R1**, listed in the reproducibility record, each of which brings the frozen branch down without repair. A refuted branch is not mended retroactively; it is recorded and replaced.

IV. What the theory forbids to itself

A last item, offered because it shows the discipline rather than asserting it.

If the $\mathcal{A}$ sector stores the rotational energy that frame dragging implies, one may ask what could be extracted from it. RM answers with its own parameters. The Earth's rotational energy is 2.14×10^{29} J; the fraction lodged in the gravitomagnetic channel is of order $\Omega_{\text{LT}}/\omega \simeq 2.3 \times 10^{-10}$, that is some 5×10^{19} J; and the relaxation time of the medium is bounded below by $\tau_0 > 10^{17}$ s, a bound obtained not from wishful thinking but from the undiminished amplitude of the LIGO signals. The maximal extractable power is therefore of order

$$\mathcal{P} \sim \frac{E_{\mathcal{A}}}{\tau_0} \sim 5 \times 10^2 \text{ W.}$$

Five hundred watts, for the entire planet. The mechanism is coherent with the theory and forbidden by it at any useful scale — and the statement is refutable, since extracting more would falsify τ_0 , and with it the transparency of the medium to gravitational waves.

THE CONTRACT

This document contains a conditional local quadratic closure of gravity, its non-saturated stellar reduction, a gauge-invariant strong-field candidate, a revised double-scale Black-Core branch, and a minimal two-derivative completion candidate. For $k \neq 0$, $Z > 0$, $\sigma \neq 0$, $B \neq 0$ and $\mu_A \neq 0$, CLG-1 derives the Poisson equation, Newton's pair energy, the inverse-Maxwell relational stress and two local tensor polarisations from one action and one matching, $Z\sigma^2 = 8\pi G$. N-Æ-02 derives, on the selected branch $\sigma < 0$, the conditional weak-field tuple

$$(\gamma_{\text{PPN}}, \beta_{\text{PPN}}, \alpha_1, \alpha_2) = (1, 1, 0, 0)$$

without fitting an optical, vector or second-order coefficient. The value of β follows both from the physical-areal to isotropic-radius map and from the static equations of the minimal constant-coefficient CNL-2D candidate. N-Æ-04 shows that the trace-free kinetic term makes the conformal momentum primary and that the elliptic constraint block separates $k \neq 0$ from the homogeneous kernel $\mathcal{K} = 3H(T)$. These are conditional closures: the labels-versus-gauge debt D-Æ-02 remains open; N-Æ-05 closes cadence-minimality debt D-Æ-03 conditionally under the Pure Cadence Principle; N-Æ-06 closes the nonlinear gravitational Dirac chain D-Æ-04A conditionally on the regular local branch, while D-Æ-04M remains open for explicit microscopic matter sectors whose formulation may alter the canonical structure. CNL-2D is therefore conditionally selected, not declared unique by spatial covariance alone. The Black-Core boundary flux, the OCH-1 selector from one action, a complete physical-EOS spherical calibration, radial stability and extension of the CA-1 certification beyond the declared test point, point-source ultraviolet stability, N-OPT-02 microscopic transport closure, boundary formation physics, perturbations and the observational lift also remain open.

The reader is not asked to believe any of it. The reader is asked to check it, and is given, for each statement, the section where the check can be made.

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How to read this document

Version v4.7.0 is the **Bulk Concordance Edition** of Pillar II. It preserves the complete local, optical, stellar, transport and numerical architecture of v4.6.0 while changing the status of the homogeneous/global sector. The later action-level bulk audit did exactly what v4.6.0 required: it tested HCR-1 and did not reproduce it. HCR-1/HCR-2 are therefore retained below as a dated historical branch, not repaired. Their place is taken by GHC-1, GEC-1 and HBR-1 on the reduced $U = 0$ branch, together with the conditional thermal/curvature completion that introduces $\varepsilon_{\mathcal{E}}^{\text{th}}$ and $U(X)$. Thermal-radiative loading is primary in the active stellar interpretation; redshift remains a genuine propagation observable and a possible secondary energetic contribution whose sign and quantitative share are not imposed. Boundary production remains a separate inhomogeneous process. This edition also receives the local P-CIN canonical audit as a strong-field foundation: positive convex Hamiltonian, regular momentum inversion, positive kinetic Hessian and non-kinetic lapse, with the full field constraint chain and hyperbolicity still open.

The underlying v4.2.0 architecture retains the v4.0.0 withdrawal of the former uniqueness claim and the v4.1.0 Pure Cadence closure. N-Æ-05 freezes the complete two-derivative cadence dependence before any nonlinear constraint calculation. N-Æ-06 then performs the first certification step requested by D-Æ-04: it introduces the universal matter interface, computes the local nonlinear scalar constraint chain, and adds the minimal covariant continuation of the already constrained antisymmetric sector. On the regular $k \neq 0$ elliptic branch, the bookkeeping remains six first-class plus ten second-class constraints, leaving two local gravitational degrees of freedom. D-Æ-04 is therefore split: D-Æ-04A is closed conditionally for the gravitational chain, while D-Æ-04M remains open and narrowed to explicit microscopic matter sectors that may contain metric/triad velocities. No cadence function or new coefficient is introduced to obtain this survival.

RM makes three different kinds of statement, and the typography of this edition keeps them distinct without interrupting the narrative:

- **RM states** introduces an ontological postulate of the framework;
- **within the RM action** introduces a mathematical consequence of the action and assumptions written in the document;
- **the calculation is open** identifies a precise quantitative equality offered to independent contradiction.

In accordance with the canonical definitions of the Constitution (V6, ch. VI), the symbol E is reserved for the state of maximal cohesion; generic energetic quantities are denoted $\mathcal{E}$ (for example $\mathcal{E}_M$, $\mathcal{E}_{\mathcal{E}}$, and $\mathcal{E}_{\text{total}}$).

THE PHYSICAL POSITION

General Relativity remains the successful macroscopic language of gravitational phenomena. RM moves beneath geometry: it asks what physical continuum must exist for gravitational geometry to appear, and what energetic account pays for the kinetic energy acquired by falling matter.

The name “Silentium” denotes an ideal limit external to the manifested Universe: no continuum, no propagation medium, and no physical support. State $\mathcal{A}$, by contrast, is the physical continuum of the manifested Universe. The two must never be confused.

WHAT STATE $\mathcal{A}$ IS NOT

State $\mathcal{A}$ is not the luminiferous aether of nineteenth-century physics. It is not an invisible gas; not a mechanical substance external to matter and light; and not a privileged frame of rest against which motion could be measured absolutely. It is a fundamental state of energy, on the same ontological level as matter and radiation. The earlier programme designation “Æther D” is withdrawn from all new work. The affirmative definition given throughout this document remains primary: this clause exists only to close a historical misreading before it begins.

Revision record — v4.7.0

September 23, 2026 — Bulk Concordance Edition.

This release applies the non-repair rule to Pillar II itself. The homogeneous HCR branch introduced in v4.6.0 was explicitly provisional and carried its own death clause: a common global action had to reproduce HCR–1 or replace it. The later homogeneous action audit did not reproduce HCR–1. The branch is therefore archived without refitting its pressure law, sign or coefficients.

The active reduced homogeneous closure is now

$$P^{(3)}R = \rho_m + u_\gamma + \varepsilon_z + u_0, \quad (\text{GHC-1})$$

$$\rho_m + 2u_\gamma - 2u_0 = 2\varepsilon_z + 3\varepsilon'_z, \quad (\text{GEC-1})$$

with

$$B(e) = 2\varepsilon_z(e) + 3\varepsilon'_z(e),$$

and preservation under the declared matter–radiation ledger gives

$$\boxed{H [3B'(e) + 3\rho_m + 8u_\gamma] = s.} \quad (\text{HBR-1})$$

No Friedmann equation, H_0 , Ω_m or fitted homogeneous response coefficient is imported.

The thermal completion uses a distinct constitutive stock $\varepsilon_{\mathcal{E}}^{\text{th}}(e)$ and the curvature-Exclusion energy

$$X = \frac{\ell_\star^4 ({}^{(3)}R)^2}{3}, \quad U(X) = u_\star \left(1 - \sqrt{1 - X} - \frac{X}{2} \right).$$

With $B_{\text{th}} = 2\varepsilon_{\mathcal{E}}^{\text{th}} + 3(\varepsilon_{\mathcal{E}}^{\text{th}})'$ and $\dot{X} = -4HX$, the corresponding conditional rate equation is

$$\boxed{H [3B'_{\text{th}} + 8XU' + 16X^2U'' + 3\rho_m + 8u_\gamma] = s.} \quad (\text{HBR-TH})$$

and reduces exactly to HBR–1 when $U = 0$.

The Maria construction is received only at its stated rank. Under the working capacity postulate UC–1, $u_{\mathcal{E}}^{\text{cap}} = P/\ell_\star^2$, it yields the conditional upper radiative load

$$u_\gamma \leq \frac{5P}{4\ell_\star^2} - u_0,$$

and its associated temperature bound. UC–1 remains a postulate. The further bridge “approach to the Maria limit $\rightarrow$ LF–09 channel closure $\rightarrow EM \rightarrow \mathcal{E}$ ” remains a working constitutive bridge, not a microscopic theorem.

The energy hierarchy is correspondingly sharpened. Thermal–radiative loading is primary in the active stellar application. Redshift remains a physical propagation observable and may contribute

secondarily to the energetic ledger, but no percentage, receiver or sign is inserted without derivation. Boundary production Γ_{∂} is a third and separate process; the unit identification $\mathcal{U}_{\partial} = P/\ell_{\star}^2$ is retained only as a conditional boundary branch, not as the residual density of newborn volume.

Finally, this edition receives the local P–CIN canonical audit. For the trace-free rate sector

$$y = \frac{\ell_{\star}^2}{4c_{\mathcal{E}}^2} \left\| \frac{K_{\text{TF}}}{N} \right\|^2 < 1,$$

the momentum map is invertible for every finite momentum, the Hamiltonian is positive and convex, the kinetic Hessian is strictly positive, and the lapse remains a Lagrange multiplier. This closes the local kinetic-health question. The complete field brackets, trace/curvature/matter coupling and hyperbolicity remain open; two physical polarisations are not claimed from the pointwise P–CIN audit alone.

Revision record — v4.6.0

September 14, 2026 — Homogeneous Exclusion-Attractor Edition.

HISTORICAL STATUS AFTER v4.7.0

This record is preserved because it documents the provisional HCR branch before its decisive action-level test. The later bulk audit did not reproduce HCR-1; v4.7.0 therefore archives the branch under its own death clause rather than recalibrating it. Equations HCR-1 through HCR-7 below remain historical results of that candidate branch only.

This release preserves every frozen local, stellar, optical and transport result of v4.5.0. It does not alter the v28 sequence, the CA-1 cross-method result, the Pure-Cadence local action, the nonlinear Dirac count or the PPN register. The revision is restricted to the homogeneous/global problem F-Æ-03.

The central change is epistemic before it is algebraic. The rate ratio

$$r \equiv \frac{H}{H_{\text{Æ}}} = \frac{p_{\text{Æ}} - \varepsilon_z(e)}{\varepsilon'_z(e)} \quad (\text{HCR-1})$$

is now declared a **provisional homogeneous constitutive relation**. It is not presented as the $k = 0$ continuation of the local Hamiltonian constraint, whose derivative block loses rank on the homogeneous mode. The future global action must reproduce HCR-1 or replace it; no sign or coefficient may be adjusted after that derivation.

For the candidate Exclusion spring

$$\varepsilon_z(e) = \kappa \frac{e}{e_{\text{cap}} - e}, \quad \kappa > 0, \quad 0 \leq e < e_{\text{cap}}, \quad (\text{HCR-2})$$

HCR-1 gives, for every finite positive pressure $p_{\text{Æ}} > 0$ held fixed on the mechanical relaxation time,

$$e_* = e_{\text{cap}} \frac{p_{\text{Æ}}}{\kappa + p_{\text{Æ}}} < e_{\text{cap}}. \quad (\text{HCR-3})$$

For a general load $p_{\text{Æ}}(e)$, the equilibrium is locally attractive on an expanding branch when

$$p'_{\text{Æ}}(e_*) < \varepsilon'_z(e_*). \quad (\text{HCR-4})$$

The capacity surface is therefore not interpreted as a point that the regular finite-load branch must reach and then be forbidden to cross. The constitutive response can organise a stable admissible state before the capacity. The repulsive reading of the capacity itself is stated only under the stronger regular-load condition that $p_{\text{Æ}}$ does not outgrow the spring energy near e_{cap} ; the weaker condition $p_{\text{Æ}} = o[(e_{\text{cap}} - e)^{-2}]$ guarantees only $r \rightarrow 0$.

The revision also records the moving-attractor correction. When $p_{\text{Æ}}$ evolves, exact $r = 0$ is

replaced in the quasistatic tracking regime by

$$r_{\text{track}} \simeq \frac{\dot{e}_*}{3H_{\mathcal{E}}} = \frac{e_{\text{cap}}}{3} \frac{\kappa p_{\mathcal{E}}}{(\kappa + p_{\mathcal{E}})^2} \frac{\dot{p}_{\mathcal{E}}/p_{\mathcal{E}}}{H_{\mathcal{E}}}. \quad (\text{HCR-5})$$

Since $\kappa p/(\kappa + p)^2 \leq 1/4$ for positive p and κ ,

$$|r_{\text{track}}| \lesssim \frac{e_{\text{cap}}}{12} \left| \frac{\dot{p}_{\mathcal{E}}/p_{\mathcal{E}}}{H_{\mathcal{E}}} \right|. \quad (\text{HCR-6})$$

Thus the attractor may possess a finite dynamical thickness; exact arrest is the stationary or adiabatic limit, not a universal instantaneous identity.

Finally, using the already frozen volume ledger

$$H_{\mathcal{E}} = H + g_{\partial}, \quad g_{\partial} = \Gamma_{\partial}/(3V_{\mathcal{E}}),$$

HCR-1 gives at exact equilibrium

$$r = 0 \implies H_{\mathcal{E}} = g_{\partial} = \frac{\Gamma_{\partial}}{3V_{\mathcal{E}}} = \frac{\mathcal{K}_{\text{cryst}} \mathcal{P}_{\gamma, \partial}}{3V_{\mathcal{E}}}. \quad (\text{HCR-7})$$

At that point the manifested-volume growth is entirely boundary production; the already manifested interior carries no additional homogeneous dilation. This is a conditional consequence of HCR-1 plus the accepted boundary ledger, not yet a derivation of the observed cosmic expansion history.

The v4.6.0 death clause is explicit: if a common global action fails to reproduce HCR-1, gives the stored reservoir a different sign, removes the sub-capacity equilibrium, requires a coefficient fitted from H_0 or an external cosmological curve, or forces double counting between redshift storage and boundary absorption, this homogeneous branch is replaced rather than repaired.

Revision record — v4.5.0

September 11, 2026 — Cross-Method and Photonic Cadence Consolidation Edition.

This release preserves the v4.4.1 cumulative cadence candidate and all earlier frozen actions, coefficients, EOS choices and v28 coordinates. It changes two technical statuses without repairing history.

First, CA-1 receives a post-v28 cross-method confirmation at the declared test pressure $p_c = 0.5028$. DOP853 shooting gives

$$R = 2.08849775, \quad M = 0.84685706,$$

while a separately implemented two-domain collocation gives

$$X_c = 0.9109684074, \quad Q_c = -0.0598152950, \quad R = 2.0884976108, \quad M = 0.8468571424,$$

with $\max |H| < 8 \times 10^{-14}$ in both domains. The two post-v28 methods reproduce the same regular $Q_c < 0$ solution and its smooth internal $Q = 0$ crossing. The earlier v28 coordinates remain a dated numerical record; they are not overwritten. The cross-method result confirms existence and regularity of the branch at this test point, not the complete high-compactness continuation, radial stability, a physical neutron-star EOS, kilometre/ $M_\odot$ calibration or external human reproduction.

Second, Book II receives N-OPT-01.1 as a conditional transport theorem. With the local Fick generator

$$\mathcal{L}_{\text{loc}} f = \nabla_i \left(D_{\text{loc}} \nabla^i f \right), \quad d\tau_{\text{cad}} = N dT,$$

the mean first-passage time expressed on the universal line T obeys

$$\boxed{\mathcal{L}_{\text{loc}} \Theta = -\frac{1}{N}.$$

On a frozen $\{g_{ij}, D_{\text{loc}}\}$ background,

$$\boxed{\frac{\Theta_N}{\Theta_{N=1}} = \left\langle \frac{1}{N} \right\rangle_{K dV_{\text{prop}}}}, \quad \min \frac{1}{N} \leq \frac{\Theta_N}{\Theta_{N=1}} \leq \max \frac{1}{N}.$$

The theorem separates matter microphysics (D_{loc}), relational spatial structure (g_{ij}) and cadence (N) without an optical fit. N-OPT-02 remains the falsification frontier for a dense, dispersive, moving microscopic medium and for a complete stellar transport solution.

The Zenodo lineage and DOI policy are unchanged; this is a new version of the same Pillar II / Book II record.

Revision record — v4.4.1

September 10, 2026 — Cumulative Saturation Closure Edition.

This release retains the v4.4.0 Revised Schwarzschild Relation and restores an explicit cumulative-saturation candidate immediately after it:

$$N_{\text{RM}}(r) = \frac{P\sqrt{f(r)}}{Q(r)} \exp\left[-\int_r^\infty \mathcal{S}(s) ds\right].$$

The new Eq. (8.6b) is deliberately not advertised as a completed derivation. It separates the local cadence contribution $\sqrt{f(r)}$ from a cumulative saturation-memory factor and keeps the definitions of $\mathcal{S}$, the boundary conditions, the admissible domain and the strong-field matching on the calculation frontier. The DOI is unchanged; v4.4.1 is a new version of the same Zenodo record.

Revision record — v4.4.0

September 10, 2026 — Revised Schwarzschild Relation Edition.

This release preserves the scientific content of v4.3.0 and adds a named editorial and physical signature for Book II:

$$\mathcal{C}_{\text{RM}}(r) = \sqrt{1 - \frac{2GM}{rc^2}} = \sqrt{1 - C(r)}.$$

The relation is explicitly presented as the RM cadence reinterpretation of the Schwarzschild observable factor: *Time does not slow down. Matter changes cadence.* No claim of completed strong-field derivation is added. The existing domain restrictions, PPN frontier, nonlinear completion requirements and falsification clauses remain in force.

Revision record — v4.3.0

Revision item	Status in v4.3.0
Canonical terminology	The former interpretive name “Principle of Saturation” is replaced in current prose by Exclusion Interaction . The historical equation label PS-0 and the mathematical word <i>saturation</i> remain where they identify an older derivation or the limit $X \rightarrow 1^-$. No new force or coupling is introduced by the rename.
Constitutive length	The role of $\ell_\star$ is made explicit: it enters the geometric invariant before any stellar EOS is chosen and therefore defines the scale at which the nonlinear RM response becomes order unity. No kilometre calibration is inferred from the reduced stellar sequence.
N-EMPR spherical bridge	The v26 hydrostatic closure, the undivided v28 cadence equation, $\partial H/\partial a = -2Q$, the two solutions at $p_c = 0.5028$, the local fold reconstruction and the accepted high-branch endpoint are imported as a dedicated technical interface. v28 is frozen; no numerical point is changed by this book revision.
N-EMPR-09 pre-audit	A separately implemented DOP853 shooting calculation is recorded only as a pre-audit. It finds a smooth interior $Q = 0$ crossing at $r_Q = 0.5148433$ and a quasi-homogeneous $Q < 0$ core, but its $Q_c < 0$ radius $R = 2.08849775$ differs from the frozen v28 value $R = 2.046754$. The discrepancy remains open and is not resolved by selection or averaging.
CA-1	A five-item solver-comparison protocol is appended as a candidate numerical certification interface: surface data, $\chi \geq 1$, separated surface/asymptotic masses, equation closures, and interior anatomy. It is an audit protocol, not a new physical law.
Audit status	No external human numerical reproduction is claimed. The v27–v28 line has adversarial checks and separately implemented calculations; the missing human reproduction remains an explicit boundary.

Revision record — v4.2.1

Maintenance item	Correction incorporated in this edition
Figure 1 — CLG-1 schematic	The former projector-based trace-mode graphic is retired. It is replaced by a schematic of the retained local chain $\mathcal{G}[q] \rightarrow \Phi[q] \rightarrow \Psi_{\mathcal{E}} \rightarrow \nabla^2 \Psi_{\mathcal{E}} = 4\pi G \rho_m \rightarrow E_{12}$ under the single matching $Z\sigma^2 = 8\pi G$. The figure explicitly separates the demonstrated local ledger $d(K + E_{12}) = 0$ from the still-open bridge to the universal reservoir; no imposed zero-mode projector appears.
Figure 2 — radiative checkpoint	The radiative-constitutive cosmology figure is restored from its source file. Its existing caption and caveat are retained: it is a reproducible numerical checkpoint, not a derived constitutive law, and the plotted history predates the ontology correction from stretching to boundary production.
Scientific content	No equation, numerical result, coupling, PPN value, Dirac rank, local degree count, falsification clause or status classification is changed by v4.2.1. The v4.2.0 result remains D-Æ-04A closed conditionally and D-Æ-04M open and narrowed.
Public identity	Cover version, PDF metadata and distributable filenames are advanced to v4.2.1 so the figure-integrated maintenance edition is bibliographically distinguishable from v4.2.0.

Revision record — v4.2.0

Audit	Correction incorporated in this full edition
N-Æ-06 — universal matter interface	The cadence map is extended to a generic local microscopic matter Lagrangian through $\mathcal{D}_T = N^{-1}(\partial_T - \mathcal{L}_B)$. Its Legendre transform is affine in N and B_i , so the unit matter current/stress coupling used in N-Æ-02 follows without independent coefficients g_B or g_Q .
Nonlinear scalar Dirac block	The frozen CNL-2D action gives primaries $p_N \approx 0$, $p_i^B \approx 0$, $\pi \approx 0$ and secondaries $\mathcal{H}_0 \approx 0$, $\mathcal{H}_i \approx 0$, $\mathcal{C} = \mathfrak{L}[N] \approx 0$. The scalar second-class principal determinant is $[2P\sqrt{g}g^{ij}k_ik_j]^4$, so the block has rank four for $k \neq 0$ on the regular elliptic branch.
Constrained antisymmetric completion	The already frozen CLG-1 coefficient μ_A admits the minimal covariant continuation $S_A^{\min} = -(\mu_A/4) \int N\sqrt{g} D_ka_{ij}D^ka^{ij}$. It carries no time derivative and therefore preserves three antisymmetric second-class pairs without adding a new parameter.
Full local degree count	The nonlinear local relational phase space has thirteen configuration variables, six first-class constraints and ten second-class constraints. On $k \neq 0$, $\mu_A \neq 0$ and the regular spectral branch, $N_{\text{grav}} = (26 - 12 - 10)/2 = 2$.
D-Æ-04	Narrowed. The nonlinear gravitational Dirac chain survives conditionally. Explicit microscopic validation remains open for matter sectors whose formulation may contain metric/triad velocities, together with strong-field spectral stability. No cadence function or new coefficient has been introduced to obtain the survival.

Revision record — v4.1.0

Audit	Correction incorporated in this full edition
N-Æ-05 — Pure Cadence	The cadence-universality rule already used in N-Æ-02 is extended to the relational sector. The local process parameter is written $d\tau_{\text{cad}} = N dT$ to avoid reifying it as a second physical time.
Conditional cadence-minimality theorem	Writing the frozen local relational density in proper-cadence units and translating it to the universal line T fixes $A = P/(4Nc_{\text{Æ}}^2)$, $B = P/(12Nc_{\text{Æ}}^2)$ and $C = PN$. No PPN value enters the derivation.
Pure Cadence Principle (PCP)	PCP states that N is a rate converter and carries no autonomous local two-derivative stored energy; therefore $D(N) = V(N) = 0$. This is a new explicit constitutive principle, not a theorem of spatial covariance or of the four-state ontology alone.
D-Æ-03	Closed <i>conditionally</i> : the former infinite functional ambiguity is reduced to one falsifiable constitutive yes/no statement. If microscopic matter, nonlinear Dirac closure or observation independently requires another cadence function, PCP is withdrawn and D-Æ-03 reopens.
D-Æ-04	Becomes the immediate certification frontier: compute the complete nonlinear Dirac chain with a universal microscopic matter interface using the cadence functions frozen by N-Æ-05.

Revision record — v4.0.0

Audit		Correction incorporated in this full edition
A-DSB-1-01 Aug. 2026)	(26	A dimensional audit found that the surface quantity $\mathcal{B}_{\text{BI}}$ as defined by Eq. (110) has units of energy, whereas $8\pi r_c \tau_\Sigma$ has units of force. The formerly printed global equation $\mathcal{B}_{\text{BI}}(r_c) + 8\pi r_c \tau_\Sigma = 0$ and its claimed equivalence with the local pressure jump are withdrawn. Equation (109) remains dimensionally homogeneous. No replacement global balance is asserted before the finite-wall variation is derived. The bulk identities and the candidate scaling $r_c \propto M^{1/3}$ are not refuted by this dimensional finding alone; the DSB-1 surface closure is open.
Conditional PPN closure		The consolidated N-Æ-02 retains the optical result $\gamma_{\text{PPN}} = +1$ on $\sigma < 0$, adds the universal sourced matter coupling, and derives $\alpha_1 = \alpha_2 = 0$ on that minimal source branch. The physical-areal to isotropic-radius map gives $\beta_{\text{PPN}} = 1$. None of the four values is fitted. Status T1 : conditional on D-Æ-02, the microscopic matter action and cadence minimality.
Double-radius audit		The discarded value $\beta_{\text{PPN}} = 0$ resulted from comparing $N^2 = 1 - C_A$, written with the physical areal radius, directly with the PPN series written in isotropic radius. The conversion $C_A = C_\rho - \frac{1}{2}C_\rho^2 + O(C_\rho^3)$ restores the second-order coefficient without changing the cadence law.
N-Æ-04 audit	V2 cross-	The exact static solution also gives $n_{\text{min}} = \Omega^2/N = (1+u)^3/(1-u)$. It reproduces $n = 1 + C_\rho + O(C_\rho^2)$ and diverges at the same finite surface where $N = 0$ and $C_A = 1$. This statement is exact within the minimal branch, not unconditional across the full functional family.
Minimal candidate	CNL-2D	The corrected action contains the required factor $c_{\text{Æ}}^{-2}$ in its trace-free kinetic term. At $\lambda = 1/3$ the trace momentum $\pi = g_{ij}\pi^{ij}$ is primary. The candidate recovers CLG-1 at quadratic order and its static vacuum equations yield the same $\beta_{\text{PPN}} = 1$. Status T0/T1 : candidate plus conditional consequences.
Audit of uniqueness		The assertion that CNL-2D is the unique two-derivative completion is withdrawn. Since N is physical under the adopted spatial gauge, the invariant action may contain functions $A(N)$, $B(N)$, $C(N)$, $D(N)$ and $V(N)$. CLG-1 fixes only the background data visible at quadratic order. In particular, the absence of $ \nabla\Lambda ^2$ fixes $D(1) = 0$, not the identity $D(N) \equiv 0$.
Conditional chain	Dirac	The primary conformal constraint and the schematic secondary chain give two local tensor modes if the nonlinear elliptic second-class block has the expected rank and the chain terminates. The full nonlinear bracket matrix with matter has not been computed. The two-mode statement is therefore not promoted beyond T1.
Homogeneous kernel		The elliptic bracket carries a factor k^2 and loses rank at $k = 0$. The linearised chain permits $\mathcal{K} = 3H(T)$ with H spatially constant and locally undetermined. This locates, but does not solve, the junction to the boundary-production cosmology.

Continued on the next page

Audit	Correction incorporated in this full edition
New priority debts	D-Æ-03 asks for a cadence-minimality theorem or a classification and exclusion of all additional admissible functions of N . D-Æ-04 asks for the complete nonlinear Dirac matrix with the microscopic matter action. Failure withdraws the affected PPN closure rather than authorising a refit.

Revision record — v3.11.0

Audit	Correction incorporated in this full edition
Cassini optical closure (new in v3.11.0)	Note N-Æ-02 derives the principal eikonal symbol $\mathfrak{P} = \omega^2/(N^2 c_{\mathbb{E}}^2) - g^{ij}[q]k_i k_j$ from relabelling covariance, electromagnetic gauge invariance and the Exclusion Interaction (PS-0), with $N^2 = 1 - C$ already registered. It gives $n^2 = (1 + \psi)/(1 - C)$ and $\gamma_{\text{PPN}} = \psi/C = -\text{sgn}(\sigma)$. Because $\psi = C$, RM admits only $\gamma_{\text{PPN}} \in \{-1, +1\}$: there is no continuous optical parameter to fit. The branch $\sigma < 0$ gives $\gamma_{\text{PPN}} = +1$ exactly, 0.91σ from the Cassini central value, and a solar-limb deflection of $1.75119''$. Status T1: derived, conditional on D-Æ-02 and on the constitutive cadence universality N-Æ-02.3. This row supersedes, and does not replace, the row “Cassini optical frontier” recorded below for v3.10.1.
Audit A-Æ-02 (new in v3.11.0)	The exclusion of $\gamma_{ij} = \delta_{ij} + q_{ij}$ stated in Section XII of v3.10.1 is correct but was prefaced by <i>for</i> $\sigma > 0$, a restriction also carried by register entry N-Æ-01.2, by Eq. (32) and by the founding-statement list. Exhaustive search of v3.10.1 shows that $Z\sigma^2 = 8\pi G$ fixes σ^2, the weak normalisation fixes σ, and $\Psi_{\mathbb{E}}$, u_{rel}, Π_{ij}, E_{12}, ε_M and the Dirac count depend on σ only through σ^2 or σ: no statement fixed the sign of σ. The structural reason is the exact discrete redundancy $(q, B, \Lambda, \sigma) \rightarrow (-q, -B, -\Lambda, -\sigma)$ of the pure relational sector, broken only when light is coupled. Status: registered, append-only; the Section XII wording is not rewritten.
Debt D-Æ-02 (new in v3.11.0)	CLG-1 §II states that spatial labels carry no physics, while the founding ontology states that $\Omega_{\mathbb{E}}$ is a physical body. Lemma 2 of N-Æ-02, and with it the uniqueness of the coupling coefficient, the value $\gamma_{\text{PPN}} = \pm 1$ and the degree count $N_{\text{phys}} = 2$, all rest on the first reading. RM cannot hold both. Status: open conceptual debt awaiting arbitration; no result of v3.11.0 may be quoted as unconditional until it is settled.
CLG-1 local closure	The spatial gauge invariant $\mathcal{G}[q]$, the multiplier constraint and the matching $Z\sigma^2 = 8\pi G$ replace the former projector and independent $\delta\Gamma$ action. Status T1: conditional theorem of the local quadratic candidate action for $k \neq 0$.
N-Æ-01 stellar reduction	For $q_{ij} = \psi\delta_{ij}$, $\mathcal{G}[q] = 2\nabla^2\psi$ and $\psi = \Phi[q]$ for $k \neq 0$. On the independently normalised weak-field branch, $\psi = -\text{sgn}(\sigma)C$. CLG-1 also gives $u_{\text{rel}} = - \nabla\Psi_{\mathbb{E}} ^2/(8\pi G)$ and an inverse-Maxwell stress with radial compression and tangential tension. Status T1/T2: algebraic identity plus conditional branch statements.
Non-saturated solar regime	With standard solar inputs and $\ell_{\star} = \ell_P$, $X_{\mathcal{R}}^{\text{core}} \simeq 7.1 \times 10^{-181}$. The former stellar $\mathbb{E} \rightarrow E$ rupture mechanism is therefore excluded on this branch. The result assigns a mechanical role to State $\mathbb{E}$ inside a star but does not replace standard stellar microphysics.

Continued on the next page

Audit	Correction incorporated in this full edition
Cassini optical frontier	The document derives $g_{00}^{\text{eff}} = 1 - C$ but no spatial optical coefficient. Identifying $\gamma_{ij} = \delta_{ij} + q_{ij}$ with an optical metric gives the wrong weak-field sign and is forbidden. The spatial part of g^{eff} must be obtained from the principal symbol of the propagation operator. The target is $\gamma_{\text{PPN}} - 1 = (2.1 \pm 2.3) \times 10^{-5}$. Status open: priority calculation, no fitted optical coefficient admitted.
CNL-BI1 nonlinear candidate	The relational curvatures of γ_{ij} replace raw components of Q_{ij} as admissible nonlinear observables. The minimal invariant $X_{\mathcal{R}} = \ell_{\star}^4 \mathcal{R}_{ij} \mathcal{R}^{ij}$ is inserted through $u_{\star}[1 - \sqrt{1 - X_{\mathcal{R}}} - X_{\mathcal{R}}/2]$. The subtraction removes the entire term linear in $X_{\mathcal{R}}$, so the correction starts at $O(q^4)$ and leaves CLG-1 unchanged at quadratic order. Status T0/T1: candidate action; low-field protection derived, nonlinear closure open.
Black-Core regularization	The response $\partial \mathcal{U}_{\text{BI}} / \partial X_{\mathcal{R}}$ diverges as $X_{\mathcal{R}} \rightarrow 1^-$ while the nonlinear density tends to $u_{\star}/2$, defining a constitutive barrier. This is a mechanism candidate for central regularisation, not yet a Black-Core solution. A local curvature threshold scales generically as $M^{1/3}$ and therefore does not by itself derive the Schwarzschild scaling M .
Double-scale raccord	The apparent M versus $M^{1/3}$ mismatch is retained as two physical radii with different operators. R_S is the optical and transport threshold; it is not the mechanical $\mathcal{A} \rightarrow E$ surface. The local curvature barrier $X_{\mathcal{R}}(r_c) = 1$ defines the mechanical phase boundary. The interval $r_c < r < R_S$ remains State $\mathcal{A}$ under nonlinear relational stress. Status T1: conditional architecture, not yet a solved wall.
Incompressible-volume scaling	Since $r_c^3 \propto M$, the ratio M/r_c^3 is universal. Within the spherical reduction this is precisely the scaling of a three-dimensional incompressible E volume. This equivalence is derived; the microscopic equation of state and the complete regular solution remain open.
Exact Born-Infeld bulk integrals	For $X_{\mathcal{R}} = (r_c/r)^6$, the energy integral is $(\pi - 3)/6$ and the magnitude of the algebraic scalar-stress integral is $(5 - \pi)/6$. Both scale as $u_{\star} r_c^3 \propto M$, and their volume-normalized coefficients sum exactly to one. Boundary derivatives of $F'(X)$ are excluded from this bulk statement and are treated separately at r_c .
Residual-memory closure	With $M_{\text{res}} = \varepsilon_M M$ and a regular core of radius r_c , the mass cancels from ρ_{res} . Matching the isotropic saturated-core density to the CLG-1 coupling gives $\varepsilon_M = 2\sqrt{\pi G Z} / c_{\mathcal{E}}^2$. Status T1: conditional on the uniform-core, isotropy and saturation assumptions.
Residual admissibility	The physical condition $0 \leq \varepsilon_M \leq 1$ imposes $0 < Z \leq c_{\mathcal{E}}^4 / (4\pi G)$. Under the independently adopted weak-field normalization $ \sigma = 16\pi G / c_{\mathcal{E}}^2$, the branch gives $Z = c_{\mathcal{E}}^4 / (32\pi G)$ and $\varepsilon_M = \sqrt{2}/4$. The numerical value is conditional on that normalization; it is not used to choose it.
OCH-1 source separation	The exterior tide and r_c respond to the complete macroscopic mass M , while the core scalar constraint responds only to $M_{\text{res}} = \varepsilon_M M$. The nonlinear State $\mathcal{A}$ sector of Zone II and the cohesive core remain in the global mass and radial-stress ledger but do not acquire an independent multiplier charge on this branch. Status P: explicit constitutive hypothesis with named rejection tests.

Continued on the next page

Audit	Correction incorporated in this full edition
Radial-pressure frontier	The local thin-wall condition $[\Pi^r_r]_{r_c^-}^{r_c^+} = 2\tau_\Sigma/r_c$ is dimensionally homogeneous and remains the pressure-jump target. Audit A-DSB-1-01 withdraws the formerly printed global expression $\mathcal{B}_{\text{BI}}(r_c) + 8\pi r_c \tau_\Sigma = 0$: with $\mathcal{B}_{\text{BI}}$ defined by Eq. (110), it adds an energy to a force and is not equivalent to the local pressure condition. The finite-wall variation must derive the correct global object and its mass scaling; τ_Σ may not be fitted to recover r_c .
Dirac matrix	The second-class block has $\det \Omega_{\text{SC}} = \frac{B^2 \mu_A^6}{16} (k^2)^{10}$ and rank ten when $B\mu_A k \neq 0$. Together with six first-class constraints it leaves two local tensor polarisations. Status T1 : the nonlinear algebra is open.
Local energy	The same matching produces $E_{12} = -Gm_1 m_2 / r$ and $d(K + E_{12}) = 0$. No local variation of u_0 is used. Point-source self-energies and ultraviolet stability remain open.
Local Horizon Dematerialization	The local field equation is solved under the stated ansatz and $g_{00} = 1 - C(r)$ is obtained. The Schwarzschild surface appears as the $C(R_S) = 1$ threshold for isolated sources. Status T1 : conditional on the RM identification of $C(r)$; reproduces the standard first-order metric function rather than replacing it.
Hawking Thermodynamics	The identity $T_H = \hbar c_{\mathcal{A}} \nabla C(R_S) / (4\pi k_B)$ is retained at the optical threshold. It is no longer identified with the latent heat of the mechanical $\mathcal{A} \rightarrow E$ transition, which occurs at r_c . Any thermal interpretation at R_S must be derived from optical or transport saturation. Status T1 : consistency identity, not an independent derivation.
Cosmic Memory (Entropy)	Bekenstein–Hawking entropy is recovered from the first law once T_H is expressed through $\nabla C(R_S)$. Status T1 : consistency result. The unitarity problem is untouched — see the closing box of Chapter 9.
Primordial Background Protection	Historical background-limit checkpoint of the former effective cosmology; not a derivation of the current HBR/thermal branch. Perturbative decoupling remains open.

Introduction

Every falling object poses a physical question so familiar that it is easily overlooked:

Where does its newly acquired kinetic energy come from?

General Relativity predicts the fall with extraordinary precision. RM asks what pays for it.

RM states that energy is conserved and exists in four fundamental physical states:

$$\{m, \text{EM}, \mathcal{A}, E\}.$$

Matter (m) concentrates energy; the electromagnetic state (EM) propagates it; State $\mathcal{A}$ distributes and transmits it through the continuum; the state E marks maximal macroscopic cohesion.

State $\mathcal{A}$ is a fundamental state of energy, on the same ontological level as matter and radiation. Gravity is the energetic relation between masses transmitted through this physical continuum; geometrical gravitation is retained as the successful macroscopic description of that manifestation, not as its substance.

FOUNDATIONAL KINEMATICS OF RM

RM does not posit a four-dimensional spacetime manifold as a physical object. Its fundamental variables live on the open universal time line and in the physical three-dimensional domain occupied by State $\mathcal{A}$:

$$T \in \mathbb{R}, \quad \mathbf{x} \in \Omega_{\mathcal{A}}(T) \subset \mathbb{R}^3.$$

The constitutive fields are written on this domain and the fundamental equations use ∂_T together with the spatial derivatives ∂_i . The tensor $Q_{ij}(T, \mathbf{x})$ is a *relational state tensor*: it records local relations carried through State $\mathcal{A}$, not a deformation of space and not a deformation of the fundamental density u_0 .

No four-dimensional integration measure, spacetime coordinate or fundamental spacetime metric is used in the local closure. Every action is written explicitly as $\int dT d^3x$. Optical or geometrical quantities introduced later are macroscopic representations, not fundamental coordinates of RM.

The demonstrated local ledger is written before any elaboration:

THE ENERGY OF A FALL

$$d(K_1 + K_2) = -dE_{12}, \quad E_{12}(r) = -\frac{Gm_1m_2}{r}.$$

Matter does not manufacture the energy of its fall. The changing relational energy supplies the kinetic gain while the fundamental density u_0 remains homogeneous.

The chapters that follow build the physical architecture from the founding symmetry, through the Cosmic Core, expansion and cohesion, to the constrained relational tensor that produces Newton's potential.

The route is direct:

1. write matter and State $\mathcal{A}$ as complementary energetic states;
2. identify a finite state of maximal cohesion and the Cosmic Core;
3. follow the radiative transfer and boundary production that drive expansion;
4. isolate the local gauge-invariant relational scalar created by masses;
5. recover the Poisson equation and the interaction energy $-Gm_1m_2/r$;
6. separate the demonstrated local mechanical ledger from the still-open universal reservoir interpretation;
7. place the nonlinear completion on the table for calculation and contradiction.

THE EDITORIAL PROMISE

A prediction is not yet a mechanism. RM identifies the continuum, the projection, the potential, and the energetic account. The equations are not being asked for permission to exist; they are being offered for contradiction.

Who Pays for Gravity? — CLG-1

STATUS OF THIS CHAPTER

CLG-1 is a conditional theorem of a local quadratic candidate action. It applies to non-homogeneous modes $k \neq 0$, with $Z > 0$, $\sigma \neq 0$, $B \neq 0$, $\mu_A \neq 0$ and decaying boundary conditions. It does not establish the nonlinear theory, the homogeneous cosmological mode, PPN closure, strong gravity, point-source self-energies or ultraviolet stability.

I. Physical separation

State $\mathcal{A}$ is a physical body with homogeneous fundamental density u_0 . It is not a geometrical fabric and it is not locally stretched by an orbit. Expansion is attributed to the production of additional State $\mathcal{A}$ at the boundary of the manifested domain. Local gravity belongs to another register: masses create relational departures carried through the already constituted body.

The field $Q_{ij}(T, \mathbf{x})$ is therefore called a *relational state tensor*. Its symmetric and antisymmetric parts are

$$q_{ij} = Q_{(ij)}, \quad a_{ij} = Q_{[ij]}. \quad (1)$$

Neither field is identified with a deformation of space or with a change in u_0 .

II. Spatial labelling gauge

The labels assigned to a local configuration carry no independent physics. The symmetric field and the shift-like auxiliary field B_i transform as

$$\delta q_{ij} = \partial_i \xi_j + \partial_j \xi_i, \quad \delta B_i = \partial_T \xi_i. \quad (2)$$

Define

$$K_{ij} = \partial_T q_{ij} - \partial_i B_j - \partial_j B_i, \quad (3)$$

$$K_{ij}^{\text{TF}} = K_{ij} - \frac{1}{3} \delta_{ij} K, \quad (4)$$

and the local spatial invariant

$$\mathcal{G}[q] = \nabla^2 \text{Tr } q - \partial_i \partial_j q_{ij}. \quad (5)$$

Direct substitution of Eq. (2) gives $\delta \mathcal{G} = 0$. For a constant q_{ij} , $\mathcal{G}[q] = 0$. The former subtraction $(1 - \mathcal{P}_0) \text{Tr } Q$ is therefore retired: the homogeneous mode is annihilated by the derivative invariant itself.

III. Local quadratic candidate action

The unique spatial Fierz–Pauli combination used in CLG-1 is

$$\mathcal{V}_{\text{FP}}^{(3)}[q] = \partial_k q_{ij} \partial_k q_{ij} - 2\partial_i q_{ij} \partial_k q_{kj} + 2\partial_i q_{ij} \partial_j q - \partial_k q \partial_k q, \quad (6)$$

where $q = \text{Tr } q$. The local action is

$$S_{\text{CLG1}} = \int dT d^3x \left[\frac{Z}{2c_{\text{E}}^2} K_{ij}^{\text{TF}} K_{\text{TF}}^{ij} - \frac{Z}{2} \mathcal{V}_{\text{FP}}^{(3)}[q] + \Lambda (\mathcal{G}[q] - \sigma \rho_m) - \frac{\mu_A}{4} \partial_k a_{ij} \partial_k a_{ij} + \mathcal{L}_A^{\text{source}} \right]. \quad (7)$$

There is no independent scalar field Φ in this action and no time derivative of a_{ij} . The multiplier Λ imposes the scalar relation; the antisymmetric sector is constrained rather than independently propagating.

The earlier local potential $\mathcal{F}_{\text{sat}}(Q)$ is deliberately absent. Its dependence on raw $\text{Tr } Q$ and on the non-invariant symmetric components breaks Eq. (2); its expansion about $Q = 0$ also contains a linear term. It cannot be added to Eq. (7). The separate candidate CNL-BI1, introduced after this quadratic closure, rebuilds nonlinear saturation from relational curvature invariants and must undergo its own Dirac analysis.

IV. The constrained scalar and Poisson’s equation

For $k \neq 0$, define only as a derived observable

$$\Phi[q] = \frac{1}{2\nabla^2} \mathcal{G}[q]. \quad (8)$$

It is not promoted to an independent field. Variation with respect to Λ gives

$$\mathcal{G}[q] = \sigma \rho_m, \quad \nabla^2 \Phi = \frac{\sigma}{2} \rho_m. \quad (9)$$

The static variation of the symmetric sector gives

$$\Lambda = Z\Phi. \quad (10)$$

The observable RM potential is then

$$\Psi_{\text{AE}} = \sigma \Lambda = Z\sigma \Phi. \quad (11)$$

One matching fixes the complete Newtonian normalisation:

$$\boxed{Z\sigma^2 = 8\pi G}. \quad (12)$$

Equations (9)–(12) give

THE POISSON LIMIT

$$\nabla^2 \Psi_{\mathbb{E}} = 4\pi G \rho_m .$$

The old zero-mode projector and the old matching $g_{\Gamma}^2/Z_{\Gamma} = 4\pi G$ are no longer used.

V. Attraction and the local ledger

Eliminating the constrained scalar for two separated sources gives

$$E_{12}(r) = -\frac{Gm_1m_2}{r} . \quad (13)$$

The same coefficient $Z\sigma^2 = 8\pi G$ fixes both the field equation and the attractive sign. The demonstrated local conservation statement is therefore

$$\mathrm{d}(K_1 + K_2) = -\mathrm{d}E_{12}, \quad \mathrm{d}(K + E_{12}) = 0 . \quad (14)$$

No local change of the homogeneous density u_0 is invoked. A future relation between this local account and the universal reservoir must be derived from the nonlinear theory; it is not part of CLG-1.

VI. General Fourier reduction

For an arbitrary non-zero wave vector $\mathbf{k}$, choose the transverse gauge $k_i q_{ij} = 0$ and write

$$q_{ij} = q_{ij}^{\mathrm{TT}} + x P_{ij}, \quad P_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j . \quad (15)$$

Then

$$\mathcal{G} = -2k^2 x, \quad \mathcal{V}_{\mathrm{FP}}^{(3)} = k^2 |q^{\mathrm{TT}}|^2 - 2k^2 x^2 . \quad (16)$$

The multiplier couples only to x and leaves q_{ij}^{TT} untouched. The two tensor modes obey the unique characteristic relation

$$\omega^2 = c_{\mathbb{E}}^2 k^2, \quad (17)$$

and have positive quadratic Hamiltonian for $Z > 0$. The constrained scalar carries the negative interaction energy required by attraction; it has no independent momentum and is not a propagating instability.

VII. Dirac matrix and degree count

The configuration variables are

$$q_{ij} \ (6), \quad a_{ij} \ (3), \quad B_i \ (3), \quad \Lambda \ (1),$$

for a total of thirteen. For $k \neq 0$, the six constraints p_{B_i} and $\mathcal{H}_i$ are first class. The ten second-class constraints form the pairs $(\mathcal{P}, \mathcal{C})$, $(p_\Lambda, \mathcal{D})$ and three copies of $(p_a, \mathcal{A})$.

Here B (without an index) denotes the non-zero coefficient appearing in the trace-constraint bracket generated by preservation of $\mathcal{P}$; it must not be confused with the auxiliary vector B_i . Its non-vanishing is a structural condition of this branch, not an additional observable fit.

With

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

their bracket matrix is

$$\Omega_{\text{SC}}(k) = \text{diag} \left(2k^2 J, \ Bk^2 J, \ \frac{\mu_A k^2}{2} J, \ \frac{\mu_A k^2}{2} J, \ \frac{\mu_A k^2}{2} J \right). \quad (18)$$

Consequently,

$$\det \Omega_{\text{SC}} = \frac{B^2 \mu_A^6}{16} (k^2)^{10}. \quad (19)$$

The factor is $1/16$, not 16 . When $B\mu_A k \neq 0$, Ω_{SC} has rank ten and

$$N_{\text{phys}} = \frac{2 \times 13 - 2 \times 6 - 10}{2} = 2. \quad (20)$$

WHAT THE COUNT MEANS

Three spatial gauge functions are sufficient because the scalar and antisymmetric sectors are removed by constitutive constraints, not by a fourth coordinate. The result is two local tensor polarisations without a fourth physical dimension.

VIII. Domain and falsification

At $k = 0$, the determinant in Eq. (19) vanishes and the derivative constraint block loses rank. The homogeneous mode belongs to the cosmological/global problem and is not a local polarisation. If $B = 0$ or $\mu_A = 0$, the Dirac algorithm must be restarted; one must not extrapolate the degree count by retaining the old classification.

The result establishes $H_{\text{TT}} > 0$ for $Z > 0$ and $E_{12} < 0$. It does not establish a lower bound for the complete Hamiltonian. Point-source self-energies diverge in the Newtonian approximation, and short-distance regularisation is an open physical requirement.

CLG-1 is refuted or replaced if a complete nonlinear action fails to preserve the spatial gauge,

if its constraint matrix changes rank on the intended branch, if it propagates additional local modes, or if it cannot reproduce the same Poisson and pair-energy normalisation without a second adjustment.

IX. The open bridge to the universal reservoir

The fundamental density remains

$$\boxed{\dot{u}_0 = 0} \tag{21}$$

in the present architecture. Local masses modify the relational state, not the mean density of State $\mathcal{A}$. Atomic clocks may still probe a constitutive dependence of material cadence on a global state variable, and radiative conversion may still feed the dynamic reservoir. Those ideas are retained, but CLG-1 does not yet derive their coupling to the local constrained field.

One action. One Newtonian matching. Two local polarisations.

State $\mathcal{A}$ Inside a Non-Saturated Star — N- $\mathcal{A}$ -01

STATUS OF THIS CHAPTER

This chapter introduces no new postulate, free parameter or constitutive relation. It is a local reduction of CLG-1, its single matching $Z\sigma^2 = 8\pi G$, its Dirac count and the already registered identification $C = -2\Psi_{\mathcal{A}}/c^2$. It asks one question: what does State $\mathcal{A}$ do inside a star that never reaches the nonlinear saturation barrier? The answer is conditional on $k \neq 0$, $Z > 0$, $\sigma \neq 0$, $B \neq 0$ and $\mu_A \neq 0$. In the historical order of the calculation, the nonlinear closure, the spatial optical operator and the PPN parameters were open here. The consolidated N- $\mathcal{A}$ -02 and corrected N- $\mathcal{A}$ -04 appended in v4.0.0 supersede that frontier conditionally; the origin-protection debts remain open.

I. Scope and terminological correction

The physical domain $\Omega_{\mathcal{A}}(T)$ fills the manifested Universe, including the interior of an ordinary star. State $\mathcal{A}$ is therefore present in stellar matter even when no mechanical $\mathcal{A} \rightarrow E$ transition occurs. Its role cannot be assigned to dispersive photon viscosity or to latent heat without contradicting the local quadratic closure.

An ordinary star is matter, state m ; it is not state E . The symbol E remains reserved for maximal macroscopic cohesion, reached on the retained Black-Core branch only at the mechanical surface r_c where $X_{\mathcal{R}} \rightarrow 1^-$. Stellar matter sources

$$\mathcal{G}[q] = \sigma\rho_m \quad (22)$$

through its matter density, not through a supposed local abundance of state E . Any earlier expression describing an ordinary star as a local concentration of E is withdrawn from the canonical vocabulary.

CORRECTION C- $\mathcal{A}$ -01 — SIGN OF THE RELATIONAL STRESS

An earlier verbal statement described State $\mathcal{A}$ as tensile along field lines and compressive transversely. The sign is reversed. On the median plane between two equal masses, $g_z = 0$ and $\Pi_{zz} = +g^2/(8\pi G) > 0$: the plane is under traction, which pulls the two halves together. The correct principal stresses are therefore *compression along the field lines and tension transversely*. The correction is append-only; the former wording is not silently rewritten.

II. Lemma 1: the isotropic representative is the constrained scalar

Take the isotropic gauge representative

$$q_{ij} = \psi \delta_{ij}. \quad (23)$$

Then

$$\mathcal{G}[q] = \nabla^2 \text{Tr } q - \partial_i \partial_j q_{ij} = 3\nabla^2 \psi - \nabla^2 \psi = 2\nabla^2 \psi. \quad (24)$$

Since the gauge-invariant scalar already defined by CLG-1 is

$$\Phi[q] \equiv \frac{1}{2\nabla^2} \mathcal{G}[q], \quad (25)$$

one obtains, for $k \neq 0$ and the same decaying boundary conditions,

$$\boxed{\psi = \Phi[q]}. \quad (26)$$

This is an algebraic identity, not a new normalisation. The inverse Laplacian is deliberately undefined on the homogeneous mode, which remains outside the local branch.

III. Lemma 2: the relational state and the cohesion field

The scalar constraint gives

$$2\nabla^2 \psi = \sigma \rho_m, \quad (27)$$

whereas the derived Poisson equation is

$$\nabla^2 \Psi_{\mathcal{E}} = 4\pi G \rho_m. \quad (28)$$

Consequently,

$$\psi = \frac{\sigma}{8\pi G} \Psi_{\mathcal{E}} = \frac{\Psi_{\mathcal{E}}}{Z\sigma}, \quad (29)$$

where the second equality uses $Z\sigma^2 = 8\pi G$. On the independently normalised weak-field branch

$$|\sigma| = \frac{16\pi G}{c_{\mathcal{E}}^2}, \quad (30)$$

one has

$$Z\sigma = \text{sgn}(\sigma) \frac{c_{\mathcal{E}}^2}{2}, \quad (31)$$

and therefore

$$\boxed{\psi = \text{sgn}(\sigma) \frac{2\Psi_{\mathcal{E}}}{c_{\mathcal{E}}^2} = -\text{sgn}(\sigma) C(r)}. \quad (32)$$

For $\sigma > 0$, the result is $\psi = -C$. This is a conditional branch identity. It does *not* license the interpretation of $\delta_{ij} + q_{ij}$ as an optical metric; that distinct question is isolated in Section XII.

IV. Lemma 3: Newtonian field energy from CLG-1

For $q_{ij} = \psi \delta_{ij}$, the four terms of the spatial Fierz–Pauli combination reduce to

Term in $\mathcal{V}_{\text{FP}}^{(3)}$	Contribution
$\partial_k q_{ij} \partial_k q_{ij}$	$+3 \nabla\psi ^2$
$-2\partial_i q_{ij} \partial_k q_{kj}$	$-2 \nabla\psi ^2$
$+2\partial_i q_{ij} \partial_j q$	$+6 \nabla\psi ^2$
$-\partial_k q \partial_k q$	$-9 \nabla\psi ^2$
Total	$-2 \nabla\psi ^2$

The Lagrangian contains $-Z\mathcal{V}_{\text{FP}}^{(3)}/2 = +Z|\nabla\psi|^2$. For a static field, the corresponding relational energy density is therefore

$$u_{\text{rel}} = -Z|\nabla\psi|^2. \quad (33)$$

Using Eq. (29),

$$u_{\text{rel}} = -\frac{|\nabla\Psi_{\mathcal{E}}|^2}{Z\sigma^2} = -\frac{|\nabla\Psi_{\mathcal{E}}|^2}{8\pi G}. \quad (34)$$

The single Newtonian matching is sufficient. No separate sign or energetic normalisation is introduced.

V. Lemma 4: hydrostatic balance is the local $\mathcal{E}$ sector

Let

$$g_i = -\partial_i \Psi_{\mathcal{E}}. \quad (35)$$

The associated inverse-Maxwell relational stress is

$$\Pi_{ij} = -\frac{1}{4\pi G} \left(g_i g_j - \frac{1}{2} \delta_{ij} g^2 \right). \quad (36)$$

Because $\nabla \times \mathbf{g} = 0$ and $\partial_i g_i = -4\pi G \rho_m$, direct differentiation gives

$$\partial_j \Pi_{ij} = \rho_m g_i. \quad (37)$$

In spherical symmetry the principal components are

$$\Pi_{\parallel} = -\frac{g^2}{8\pi G} \quad (\text{radial compression}), \quad \Pi_{\perp} = +\frac{g^2}{8\pi G} \quad (\text{tangential tension}). \quad (38)$$

The usual hydrostatic equation $\partial_i P = \rho_m g_i$ can therefore be written as the exact local stress balance

$$\partial_j (P \delta_{ij} - \Pi_{ij}) = 0. \quad (39)$$

Within this branch, a star is not matter supported against a physical four-dimensional geometry. It is a local balance between the kinetic pressure of matter and the relational stress transmitted by State $\mathcal{A}$.

VI. Solar order-of-magnitude audit

For orientation, take

$$GM_{\odot} = 1.32712 \times 10^{20} \text{ m}^3 \text{ s}^{-2}, \quad R_{\odot} = 6.957 \times 10^8 \text{ m}, \quad \rho_c = 1.50 \times 10^5 \text{ kg m}^{-3}, \quad (40)$$

together with $P_c = 2.34 \times 10^{16} \text{ Pa}$, a standard $n = 3$ polytropic estimate, $\ell_{\star} = \ell_P = 1.616 \times 10^{-35} \text{ m}$ and $|\sigma| = 16\pi G/c^2$.

Quantity	Value	Status
$C(R_{\odot}) = 2GM_{\odot}/(R_{\odot}c^2)$	4.245×10^{-6}	direct
$C(0)$	1.81×10^{-5}	conditional, $n = 3$
$ \psi $ at the centre	1.8×10^{-5}	Eq. (32)
Core-surface cadence offset $\Delta C/2$	6.9×10^{-6}	$g_{00}^{\text{eff}} = 1 - C$
Accumulated offset over 4.6 Gyr	$3.2 \times 10^4 \text{ yr}$	derived
$ \mathcal{R} _{\text{core}} = 16\pi G\rho_c/c^2$	$5.60 \times 10^{-21} \text{ m}^{-2}$	direct
$X_{\mathcal{R}}$ at the centre, $\ell_{\star} = \ell_P$	7.1×10^{-181}	conditional $\ell_{\star}$
$\ell_{\star}$ required for $X_{\mathcal{R}} = 1$	$1.76 \times 10^{10} \text{ m}$	derived
g_{max} near $0.25R_{\odot}$	$\simeq 2.2 \times 10^3 \text{ m s}^{-2}$	external solar model
$ \Pi_{\parallel} $ there	$\simeq 2.9 \times 10^{15} \text{ Pa}$	Eq. (36)
Plasma pressure there	$\simeq 1.5 \times 10^{15} \text{ Pa}$	external solar model
Eikonal Shapiro-scale delay across $R_{\odot}$	$\sim 2.3 \times 10^{-5} \text{ s}$	order of magnitude

Two readings are retained. First, on the Planck-scale branch the nonlinear barrier is out of reach in the solar core by roughly 180 orders of magnitude. A main-sequence star is therefore non-saturated in the strict sense $X_{\mathcal{R}} \ll 1$ throughout. Second, $|\Pi_{\parallel}|/\rho_E^{\text{max}} \sim 7 \times 10^8$ is not by itself a contradiction: relational stress and cohesive energy density are different physical categories. It is nevertheless recorded as an independent warning that the adopted value of ρ_E^{max} and the anchoring budget must ultimately be derived together rather than compared rhetorically.

VII. Conditional losslessness and achromaticity

For $B\mu_A k \neq 0$, the Dirac analysis leaves only two propagating tensor modes, both with front speed $c_{\mathcal{A}}$. The scalar responding to ρ_m is constrained and has no independent canonical oscillator into which linear scalar energy can be irreversibly deposited.

This fact supports, but does not alone prove, an achromatic electromagnetic response. The precise conditional statement is the following: if the linear response of the isolated continuum contains no additional matter or dissipative channel, if its absorptive part vanishes over the full

relevant spectrum,

$$\text{Im } \chi(\omega) = 0, \quad (41)$$

and if the usual causal and high-frequency conditions required by the subtracted Kramers–Kronig relation hold, then its real part is frequency independent on that branch. The implication is one-way:

$$\boxed{\text{Dirac closure plus all-spectrum losslessness} \implies \text{achromatic linear response}}. \quad (42)$$

The stronger equivalence between the Dirac rank, achromaticity, the massless radiation equation of state and $\kappa_S = 1$ is not asserted here. It requires the explicit electromagnetic coupling operator. Nonlinear CNL-BI1 backgrounds must also be analysed separately.

VIII. Optical budget: what State $\mathcal{A}$ does not do

At quadratic order, State $\mathcal{A}$ changes the characteristic operator without supplying a frequency scale. The solar gravitational contribution to the propagation time is of order

$$\Delta t_{\mathcal{A}} \sim \frac{1}{c} \int_0^{R_\odot} C(r) dr \sim 2.3 \times 10^{-5} \text{ s}, \quad (43)$$

whereas the radiative diffusion time through the Sun is of order

$$t_{\text{diff}} \sim 10^5 \text{ yr} \simeq 3.2 \times 10^{12} \text{ s}. \quad (44)$$

The ratio is approximately 1.4×10^{17} . CLG-1 neither traps the photon for this duration nor needs to do so. Bound–free and free–free opacity and Thomson scattering belong to the EM–matter channel. State $\mathcal{A}$ transmits the relational response; matter absorbs and re-emits radiation.

IX. Five roles of State $\mathcal{A}$ in a non-saturated star

1. **It carries hydrostatic support.** Equation (39) closes the local stress balance.
2. **It regulates physical cadence.** The retained optical-time coefficient is $g_{00}^{\text{eff}} = 1 - C$; on the quoted solar model the centre–photosphere difference is about 6.9 ppm.
3. **It carries the contraction ledger.** Kelvin–Helmholtz contraction changes E_{12} with $d(K + E_{12}) = 0$ and $\dot{u}_0 = 0$; the star changes its relational configuration rather than excavating the homogeneous continuum.
4. **It responds without a scalar radiation channel.** A spherical pulsation perturbs the constrained scalar but emits no tensor mode at linear order. Calling this response a perfect spring remains conditional on the absence of nonlinear and matter-mediated damping.
5. **It contributes to the stellar thermostat through the virial relation, not through rupture.** A deeper relational well raises the virial temperature and hence the nuclear rates.

The mass–luminosity law is not rederived here.

The directional intuition of the retired Pillar II therefore survives only in a restricted form: State $\mathcal{A}$ participates in stellar regulation, but through conservative relational stress and virial temperature, not through an $\mathcal{A} \rightarrow E$ phase rupture in an ordinary star.

X. What this chapter does not establish

1. It establishes no new stellar prediction and supplies no replacement for a stellar-structure calculation.
2. It does not derive the spatial part of the effective optical operator, the full Shapiro delay, light deflection or γ_{PPN} .
3. It does not prove achromaticity on a nonlinear CNL-BI1 background.
4. It does not derive the helioseismic sound-speed profile, measured at high precision over most of the solar radius.
5. The quoted central compactness, maximum acceleration and local plasma pressure use an external standard solar model as comparison input.

XI. Rejection conditions

This chapter fails if any of the following occurs:

1. Eqs. (26) or (34) fail under direct recalculation.
2. A non-plasma chromatic residual is observed in a regime assigned to the compressed continuum, once ordinary matter and plasma effects have been removed.
3. The nonlinear Dirac analysis produces an additional dissipative mode on the physical branch.
4. The hydrostatic balance cannot be extended into compact-star regimes without a new fitted coupling.
5. An otherwise admissible value of $\ell_\star$ makes $X_{\mathcal{R}} = 1$ inside a main-sequence star, reopening the retired stellar-rupture branch.

XII. Priority frontier: the Cassini factor-of-two anomaly of RM

The anomaly named here belongs to RM, not to the Cassini data. Pillar II derives the weak optical-time coefficient

$$g_{00}^{\text{eff}} = 1 - C(r), \tag{45}$$

but it has not derived the spatial part of the eikonal operator. In standard PPN language the light-time and deflection coefficient is proportional to $1 + \gamma_{\text{PPN}}$. A time coefficient alone corresponds to the Newtonian half of the observed effect, conventionally written $\gamma_{\text{PPN}} = 0$.

The naive route is now excluded internally. For $\sigma > 0$, Eq. (32) gives

$$\gamma_{ij} = \delta_{ij} + q_{ij} = (1 - C)\delta_{ij}, \quad (46)$$

whereas the observed weak-field optical response requires the opposite first-order spatial sign. This is not a refutation of CLG-1, because γ_{ij} is a relational state tensor, not an optical spatial metric. It is a proof that the two objects may not be identified.

The spatial coefficient must therefore emerge from the eikonal reduction of the electromagnetic propagation operator on State $\mathcal{E}$. The opposed experimental target is the Cassini solar-conjunction result

$$\boxed{\gamma_{\text{PPN}} - 1 = (2.1 \pm 2.3) \times 10^{-5} .} \quad (47)$$

The measurement used multifrequency radio links to control the dispersive solar-plasma contribution.¹

CASSINI CALCULATION CONTRACT

Derive the principal eikonal symbol of the electromagnetic operator using only T , $\Omega_{\mathcal{E}}(T)$ and the admissible relational invariants. Expand it to first order in the solar field and read both the temporal and spatial optical coefficients. No identification of q_{ij} with a metric, no fourth physical dimension and no optical parameter adjusted to force $\gamma_{\text{PPN}} = 1$ are permitted. If the unique operator does not reproduce Eq. (47), the local RM branch is rejected rather than recalibrated.

This calculation precedes preferred-frame parameters: Cassini tests the leading factor of two, whereas α_1 and α_2 test smaller corrections after the optical operator has already survived.

¹B. Bertotti, L. Iess and P. Tortora, “A test of general relativity using radio links with the Cassini spacecraft”, *Nature* **425**, 374–376 (2003), doi:10.1038/nature01997.

XIII. Append-only register

Reference	Statement	Status
N-Æ-01.1	$\psi = \Phi[q]$ in the isotropic representative	T2, identity
N-Æ-01.2	$\psi = -C$ for $\sigma > 0$ on the stated weak branch	T1, conditional
N-Æ-01.3	$u_{\text{rel}} = - \nabla\Psi_{\text{Æ}} ^2/(8\pi G)$	T1, derived
N-Æ-01.4	Inverse-Maxwell stress: radial compression, tangential tension	T1, derived
N-Æ-01.5	$\partial_j(P\delta_{ij} - \Pi_{ij}) = 0$	T1, derived
N-Æ-01.6	$X_{\mathcal{R}}^{\text{solar core}} \simeq 7.1 \times 10^{-181}$ for $\ell_{\star} = \ell_P$	T1, conditional
N-Æ-01.7	Dirac closure plus all-spectrum losslessness implies achromaticity	T1, conditional
N-Æ-01.8	The former stellar-rupture branch is incompatible with the retained local branch	T1, conditional
N-Æ-01.9	The photon diffusion time belongs to the matter-opacity sector	T1, assignment from scale audit
N-Æ-01.10	Virial rather than rupture regulation of an ordinary star	T0, assignment
C-Æ-01	Correction of the relational-stress sign	registered
F-Æ-01	Spatial eikonal operator and γ_{PPN}	open, absolute priority

NON-REPAIR CLAUSE

The retired Pillar II is downgraded to T0 and removed from the submission file; it is not erased from the research history. Its surviving intuition is explicitly credited in Section IX. The sign correction C-Æ-01 coexists with the statement it corrects. No branch rejected here may be repaired retroactively.

Gauge-Invariant Nonlinear Saturation — CNL-BI1

STATUS OF THIS CHAPTER

CNL-BI1 is the first minimal nonlinear candidate built after the withdrawal of the raw potential $\mathcal{F}_{\text{sat}}(Q)$. The gauge invariance of its building blocks and the protection of the complete quadratic CLG-1 sector are derived. Its full Dirac algebra, spherical solutions, ultraviolet stability and Black-Core phase matching are open. It is therefore a candidate branch, not a completed strong-gravity theory.

I. Ontological restriction

No four-dimensional manifold, space-time metric or measure d^4x is introduced. The variables remain the universal time T and the three labels $\mathbf{x}$ of the physical body State $\mathcal{A}$. The nonlinear tensor

$$\gamma_{ij} = \delta_{ij} + q_{ij} + O(q^2) \quad (48)$$

is a *relational state tensor*: it records the internal relation of neighbouring states under a change of spatial labels. It is not a claim that physical space is a deformable substance. Under an infinitesimal relabelling,

$$\delta_\xi \gamma_{ij} = \mathcal{L}_\xi \gamma_{ij}, \quad \delta_\xi q_{ij} = \partial_i \xi_j + \partial_j \xi_i + O(q\xi). \quad (49)$$

The nonlinear completion must be assembled only from quantities transforming covariantly under Eq. (49).

II. Relational curvatures

Let D_i denote the derivative compatible with γ_{ij} . Its intrinsic relational curvature supplies

$$\mathcal{R}_{ij}[\gamma], \quad \mathcal{R} = \gamma^{ij} \mathcal{R}_{ij}, \quad \mathcal{S}_{ij} = \mathcal{R}_{ij} - \frac{1}{3} \gamma_{ij} \mathcal{R}. \quad (50)$$

At first order around Eq. (48),

$$\mathcal{R}_{ij}^{(1)} = \frac{1}{2} \left(\partial_k \partial_i q^k{}_j + \partial_k \partial_j q^k{}_i - \nabla^2 q_{ij} - \partial_i \partial_j q \right), \quad (51)$$

$$\mathcal{R}^{(1)} = \partial_i \partial_j q_{ij} - \nabla^2 q = -\mathcal{G}[q]. \quad (52)$$

Thus the scalar that produced Poisson's equation in CLG-1 is exactly the negative linear limit of the relational scalar curvature. The nonlinear trial constraint

$$\boxed{\mathcal{C}_{\text{NL}}[\gamma] \equiv -\mathcal{R}[\gamma]} \quad (53)$$

satisfies $\mathcal{C}_{\text{NL}}^{(1)} = \mathcal{G}[q]$ without a projector.

In three dimensions the full intrinsic Riemann tensor is algebraically determined by $\mathcal{R}_{ij}$. A minimal independent invariant set may therefore be taken as

$$\mathcal{R}, \quad \mathcal{S}_{ij}\mathcal{S}^{ij}, \quad \mathcal{S}_i^j \mathcal{S}_j^k \mathcal{S}_k^i. \quad (54)$$

The Cotton tensor is also admissible, but it carries an additional spatial derivative and vanishes on broad conformally flat and spherical branches. It is not used in the minimal candidate.

III. The minimal curvature norm

The first positive dimensionless curvature measure is

$$X_{\mathcal{R}} = \ell_{\star}^4 \mathcal{R}_{ij} \mathcal{R}^{ij} = \ell_{\star}^4 \left(\mathcal{S}_{ij} \mathcal{S}^{ij} + \frac{1}{3} \mathcal{R}^2 \right), \quad 0 \leq X_{\mathcal{R}} < 1. \quad (55)$$

Here $\ell_{\star}$ is a constitutive length to be fixed independently. The use of the full Ricci norm is essential. In a vacuum exterior the scalar constraint may give $\mathcal{R} = 0$ while the trace-free tidal curvature $\mathcal{S}_{ij}$ remains non-zero; a function of $\mathcal{R}$ alone would therefore miss precisely the strong local field that the completion must control.

IV. The subtracted Born–Infeld barrier

The proposed nonlinear density is

$$\mathcal{U}_{\text{BI}}(X_{\mathcal{R}}) = u_{\star} \left[1 - \sqrt{1 - X_{\mathcal{R}}} - \frac{X_{\mathcal{R}}}{2} \right]. \quad (56)$$

The subtraction $-X_{\mathcal{R}}/2$ is not cosmetic. It removes the complete term linear in $X_{\mathcal{R}}$:

$$1 - \sqrt{1 - X_{\mathcal{R}}} - \frac{X_{\mathcal{R}}}{2} = \frac{X_{\mathcal{R}}^2}{8} + \frac{X_{\mathcal{R}}^3}{16} + O(X_{\mathcal{R}}^4). \quad (57)$$

Because $\mathcal{R}_{ij} = O(q)$ and $X_{\mathcal{R}} = O(q^2)$, Eq. (56) begins at $O(q^4)$. Multiplication by $\sqrt{\gamma} = 1 + O(q)$ does not change that conclusion. Consequently CNL-BI1 introduces, around the homogeneous branch,

- no tadpole;
- no quadratic mass;
- no renormalisation of the quadratic tensor cone;
- no change to the CLG-1 bracket matrix at quadratic order.

This is the exact low-field protection condition that the retired $\mathcal{F}_{\text{sat}}(Q)$ did not satisfy.

The nonlinear response is nevertheless singular at the boundary of its domain:

$$\frac{\partial \mathcal{U}_{\text{BI}}}{\partial X_{\mathcal{R}}} = \frac{u_{\star}}{2} \left(\frac{1}{\sqrt{1 - X_{\mathcal{R}}}} - 1 \right) \xrightarrow{X_{\mathcal{R}} \rightarrow 1^-} +\infty, \quad (58)$$

while

$$\mathcal{U}_{\text{BI}} \xrightarrow{X_{\mathcal{R}} \rightarrow 1^-} \frac{u_{\star}}{2}. \quad (59)$$

The correction density is finite, but the marginal cost of increasing the relational curvature diverges. The candidate therefore defines a constitutive barrier rather than a hard clipping imposed after the equations are solved.

V. Nonlinear action template

Define the relational rate

$$\mathbb{K}_{ij} = \frac{1}{2} (\partial_T \gamma_{ij} - D_i B_j - D_j B_i), \quad \mathbb{K}_{ij}^{\text{TF}} = \mathbb{K}_{ij} - \frac{1}{3} \gamma_{ij} \mathbb{K}. \quad (60)$$

A minimal completion template is

$$S_{\text{CNL-BI}} = \int dT d^3x \sqrt{\gamma} \left[\frac{Z}{2c_{\mathcal{E}}^2} \mathbb{K}_{ij}^{\text{TF}} \mathbb{K}_{\text{TF}}^{ij} - \mathcal{V}_{\text{rel}}[\gamma] - \mathcal{U}_{\text{BI}}(X_{\mathcal{R}}) + \Lambda(-\mathcal{R}[\gamma] - \sigma \rho_m) \right] + S_A. \quad (61)$$

The functional $\mathcal{V}_{\text{rel}}$ is required to reproduce $Z\mathcal{V}_{\text{FP}}^{(3)}/2$ at quadratic order, up to a boundary term, and S_A must reproduce the constrained antisymmetric sector of CLG-1. Equation (61) is therefore a template with explicit matching conditions, not yet a unique action. It contains only the universal time T , three-dimensional relational quantities and the material source.

VI. What it can and cannot do for Black Cores

In a weak isolated exterior, the tidal norm has the order

$$\mathcal{S}_{ij} \mathcal{S}^{ij} \simeq 6 \left(\frac{GM}{c_{\mathcal{E}}^2 r^3} \right)^2. \quad (62)$$

The threshold $X_{\mathcal{R}} = 1$ then gives

$$\boxed{r_{\text{sat}} \simeq 6^{1/6} \left(\frac{\ell_{\star}^2 GM}{c_{\mathcal{E}}^2} \right)^{1/3}}. \quad (63)$$

This result supplies a candidate scale for central regularisation. Since $r_{\text{sat}} \propto M^{1/3}$ while the compactness scale is proportional to M , curvature saturation cannot be advertised as a derivation

of the same surface. DSB-1, revised in the following chapter, assigns the two thresholds to different operators: R_S is optical and transmissive, whereas $r_{\text{sat}} = r_c$ is the mechanical $\mathcal{A} \rightarrow E$ boundary. The region between them remains State $\mathcal{A}$ under nonlinear stress rather than a progressive conversion layer.

NONLINEAR VERDICT

CNL-BI1 closes one precise conceptual hole: a nonlinear saturation term can be built exclusively from gauge-covariant derivatives, can become infinitely resistant at a finite curvature norm, and can leave every verified quadratic result of CLG-1 untouched. It does not yet prove a nonsingular Black Core. The next decisive calculations are the nonlinear Dirac matrix on a background $0 < X_{\mathcal{R}} < 1$ and the regular static spherical boundary-value problem.

VII. Mandatory tests before promotion

1. Expand Eq. (61) through fourth order and verify explicitly that every quadratic CLG-1 coefficient remains unchanged.
2. Recompute the complete Dirac algorithm on a non-zero background $X_{\mathcal{R}} = X_0 < 1$; no degree count may be inherited from the quadratic branch.
3. Derive the principal symbol and verify that the tensor front remains unique and equal to $c_{\mathcal{A}}$.
4. Solve the static spherical equations with a regular centre and determine whether $X_{\mathcal{R}} < 1$ everywhere.
5. Derive the Born–Infeld boundary flux and radial pressure junction at the local mechanical surface r_c , rather than forcing the optical and mechanical radii to coincide.
6. Check point-source self-energies and the lower-bound question for regularised sources. The bounded value of $\mathcal{U}_{\text{BI}}$ alone does not prove that the complete Hamiltonian is bounded.

The Double-Scale Black Core — DSB-1

STATUS OF THIS CHAPTER

DSB-1 freezes a conditional three-zone architecture, revised in v3.10.0. Its two radii are no longer two stages of one extended conversion: R_S is the optical transport threshold and r_c is the mechanical $\mathcal{A} \rightarrow E$ surface. The law $r_c \propto M^{1/3}$ makes M/r_c^3 universal and therefore gives the E -rich core the scaling signature of a three-dimensional incompressible volume. The exact Born–Infeld bulk integrals are derived below. OCH-1 explicitly separates the complete mass sourcing the exterior tide from the residual material charge sourcing the core scalar constraint. Audit A-DSB-1-01 withdraws the formerly printed global surface equation because its two terms have different dimensions. The local pressure condition remains, but the dimensionally homogeneous global wall–flux variation and its mass scaling are open. DSB-1 remains a strong-field candidate branch, not a completed Black-Core solution.

I. Two thresholds, not one contradiction

Define the gravitational length

$$r_g = \frac{GM}{c_{\mathcal{A}}^2}. \quad (64)$$

The outer threshold is fixed by accumulated compactness:

$$R_S = 2r_g = \frac{2GM}{c_{\mathcal{A}}^2} \propto M. \quad (65)$$

The inner threshold is fixed by the local relational-curvature norm:

$$r_c = r_{\text{sat}} = 6^{1/6} \left(\ell_{\star}^2 r_g \right)^{1/3} \propto M^{1/3}. \quad (66)$$

Their ratio is

$$\frac{r_c}{R_S} = \frac{6^{1/6}}{2} \left(\frac{\ell_{\star}}{r_g} \right)^{2/3}. \quad (67)$$

For a macroscopic branch with $r_g \gg \ell_{\star}$, the ordering $r_c \ll R_S$ follows. The two scalings answer different questions: R_S marks optical and transport saturation, whereas r_c marks the mechanical curvature barrier and the phase transition into the E -rich volume.

II. The three physical zones

Zone	Radial domain	Physical role
I — transmissive exterior	$r \geq R_S$	State $\mathcal{A}$ carries the exterior relational field; $C(r) = R_S/r$ for the isolated branch. The surface R_S is an optical and transport threshold.
II — nonlinear $\mathcal{A}$ shell	$r_c < r < R_S$	State $\mathcal{A}$ remains State $\mathcal{A}$ while its relational curvature and stored stress increase toward the mechanical barrier. There is no progressive $\mathcal{A} \rightarrow E$ conversion in this zone.
III — regular core	$0 \leq r \leq r_c$	The mechanical surface satisfies $X_{\mathcal{R}}(r_c) = 1$; inside it, an incompressible E -rich volume retains the non-converted material fraction.

In Zone I,

$$C_{\text{out}}(r) = \frac{R_S}{r}, \quad C(R_S) = 1, \quad C'(R_S) = -\frac{1}{R_S}. \quad (68)$$

The thermodynamic surface therefore retains the Hawking-temperature identity

$$\boxed{T_H = \frac{\hbar c_{\mathcal{A}}}{4\pi k_B} |C'(R_S)| = \frac{\hbar c_{\mathcal{A}}^3}{8\pi G M k_B}.} \quad (69)$$

This identity belongs to the optical surface R_S . It is not generated at the central curvature barrier and is no longer interpreted as the latent heat of the mechanical $\mathcal{A} \rightarrow E$ conversion. Any RM thermal mechanism at R_S must be derived from optical or transport saturation.

Let χ be an order parameter only for the mechanical wall at r_c , with $\chi_{\mathcal{A}}$ immediately outside and χ_E immediately inside. In the thin-wall reduction,

$$\chi(r_c^+) = \chi_{\mathcal{A}}, \quad \chi(r_c^-) = \chi_E. \quad (70)$$

No interpolation from R_S to r_c is imposed. The wall profile and its thickness must follow from the local phase equation and the Born–Infeld boundary flux.

III. Exact bulk response of the nonlinear $\mathcal{A}$ shell

The mechanical interpretation makes the $M^{1/3}$ law constructive rather than defective. From Eq. (66),

$$\boxed{\frac{M}{r_c^3} = \frac{c_{\mathcal{A}}^2}{\sqrt{6} G \ell_{\star}^2},} \quad (71)$$

so the mean total mass density associated with the mechanical volume is

$$\boxed{\bar{\rho}_{\text{core}} = \frac{3M}{4\pi r_c^3} = \frac{3c_{\mathcal{A}}^2}{4\pi\sqrt{6} G \ell_{\star}^2}.} \quad (72)$$

This is the exact scaling of a three-dimensional incompressible volume. It does not by itself derive the microscopic equation of state of E .

In the exterior tidal reduction, the normalization $X_{\mathcal{R}}(r_c) = 1$ gives throughout the unsaturated part of Zone II

$$X_{\mathcal{R}}(r) = \left(\frac{r_c}{r}\right)^6. \quad (73)$$

Write

$$F(X) = 1 - \sqrt{1 - X} - \frac{X}{2}. \quad (74)$$

For $R_S/r_c \gg 1$, its bulk energy integral is exact:

$$E_{\text{BI}}^{\text{II}} = 4\pi u_{\star} r_c^3 \int_1^{R_S/r_c} y^2 F(y^{-6}) dy, \quad (75)$$

$$\int_1^{\infty} y^2 F(y^{-6}) dy = \boxed{\frac{\pi - 3}{6}}, \quad (76)$$

and therefore

$$\boxed{E_{\text{BI}}^{\text{II}} = \frac{2\pi}{3}(\pi - 3)u_{\star}r_c^3 + O\left[\left(\frac{r_c}{R_S}\right)^9\right]}. \quad (77)$$

Because $r_c^3 \propto M$, this bulk energy is automatically proportional to M without a mass-dependent coefficient.

The scalar channel does not read F alone. Under the local isotropic variation $\delta\gamma_{ij} = 2\omega\gamma_{ij}$, the Born–Infeld action gives

$$\begin{aligned} \Pi^i_i = u_{\star} \Big[& 3F - 4XF' - 2\ell_{\star}^4 \nabla_i \nabla_j (F' \mathcal{R}^{ij}) \\ & - 2\ell_{\star}^4 \nabla^2 (F' \mathcal{R}) \Big], \end{aligned} \quad (78)$$

with the sign convention of Eq. (108). The algebraic bulk magnitude has the second exact integral

$$\int_1^{\infty} y^2 [4XF'(X) - 3F(X)]_{X=y^{-6}} dy = \boxed{\frac{5 - \pi}{6}}. \quad (79)$$

The volume-normalized coefficients obey

$$\boxed{\frac{\pi - 3}{2} + \frac{5 - \pi}{2} = 1}. \quad (80)$$

Thus the finite bulk density and the algebraic scalar response form an exact complementary partition. Equation (80) is not yet the OCH-1 closure: the two double-divergence terms in Eq. (78) become a boundary flux, and $F'(X)$ diverges as $X \rightarrow 1^-$.

IV. Residual matter, source separation and disappearance of the total mass

Let the conversion leave the universal residual fraction

$$M_{\text{res}} = \varepsilon_M M. \quad (81)$$

OCH-1 — Ontological Core Hypothesis

The two occurrences of mass in DSB-1 belong to two distinct physical operators. The exterior relational tide is governed by the complete macroscopic mass,

$$M_{\text{ext}} = M = M_{\text{res}} + M_{\text{coh}}, \quad M_{\text{coh}} = (1 - \varepsilon_M)M, \quad (82)$$

where M_{coh} denotes the cohesive contribution retained in the global ledger, together with the nonlinear stress energy stored by State $\mathcal{A}$ in Zone II. Consequently, the curvature threshold uses

$$\mathcal{T}_{\text{ext}}(r_c) \propto \frac{GM}{r_c^3}, \quad (83)$$

and Eq. (66) continues to involve M , not $\varepsilon_M M$.

The local saturated-core scalar charge is postulated to be different:

$$\rho_{\text{rel}}^{(\text{core})} \equiv \rho_{m,\text{res}}, \quad -\mathcal{R}_c = \sigma \rho_{m,\text{res}}. \quad (84)$$

Equivalently, on the OCH-1 branch, the cohesive sector and the mechanical phase wall carry no independent charge under the CLG-1 multiplier,

$$\left. \frac{\delta S_{E+\Sigma}}{\delta \Lambda} \right|_{\text{core}} = 0, \quad (85)$$

while their energy and radial stress remain present in M_{ext} and in Π^r_r . OCH-1 therefore does *not* declare the nonlinear State $\mathcal{A}$ shell or state E massless, absent or dynamically inert. It declares that global exterior mass and local scalar charge are not the same observable after mechanical conversion.

STATUS OF OCH-1

OCH-1 is a constitutive source-selection hypothesis. It is not derived from CLG-1 or CNL-BI1. It is rejected if a variational completion gives state E the same multiplier charge as residual matter, if the exterior field is sourced only by M_{res} , or if the radial junction cannot close the full energy–stress ledger without an additional fitted coupling.

For a regular core of radius r_c , its mean mass density is

$$\rho_{\text{res}} = \frac{3\varepsilon_M M}{4\pi r_c^3}. \quad (86)$$

Using Eq. (66),

$$r_c^3 = \sqrt{6} \ell_\star^2 \frac{GM}{c_\mathbb{E}^2}, \quad (87)$$

and the total mass cancels exactly:

$$\rho_{\text{res}} = \frac{3\varepsilon_M c_\mathbb{E}^2}{4\pi\sqrt{6} G \ell_\star^2}. \quad (88)$$

The corresponding residual energy density is

$$u_{\text{res}} = \rho_{\text{res}} c_\mathbb{E}^2 = \frac{3\varepsilon_M c_\mathbb{E}^4}{4\pi\sqrt{6} G \ell_\star^2}. \quad (89)$$

Thus a universal residual fraction and the $M^{1/3}$ core law imply a universal residual central density.

V. Conditional determination of ε_M

In the isotropic core limit, $\mathcal{S}_{ij}\mathcal{S}^{ij} \simeq 0$ and

$$X_{\mathcal{R}} \simeq \frac{\ell_\star^4}{3} \mathcal{R}^2. \quad (90)$$

The barrier $X_{\mathcal{R}} \rightarrow 1^-$ then fixes

$$|\mathcal{R}_c| \rightarrow \frac{\sqrt{3}}{\ell_\star^2}. \quad (91)$$

Using the nonlinear continuation of the CLG-1 scalar constraint,

$$-\mathcal{R}_c = \sigma \rho_{\text{res}}, \quad (92)$$

the maximal residual density is

$$\rho_{\text{sat}} = \frac{\sqrt{3}}{|\sigma| \ell_\star^2}. \quad (93)$$

The minimal raccord identifies Eqs. (88) and (93). It gives

$$\varepsilon_M = \frac{4\pi\sqrt{2} G}{c_\mathbb{E}^2 |\sigma|}. \quad (94)$$

Finally, the already derived Newtonian matching $Z\sigma^2 = 8\pi G$ yields

$$\varepsilon_M = \frac{2\sqrt{\pi G Z}}{c_\mathbb{E}^2}. \quad (95)$$

The residual fraction is therefore not fitted independently in DSB-1. Equation (95) remains conditional on core isotropy, the uniform-density reduction, the saturated limit and the continuation of the same σ into the nonlinear branch.

Admissibility bound on the elastic normalization

Because ε_M is a residual mass fraction, its physical domain is

$$0 \leq \varepsilon_M \leq 1. \quad (96)$$

For the positive propagating normalization $Z > 0$, Eq. (95) therefore implies

$$\boxed{0 < Z \leq \frac{c_{\mathbb{E}}^4}{4\pi G}}. \quad (97)$$

This is not an additional fit: any value of Z above the bound would force $\varepsilon_M > 1$ and would exclude the DSB-1 residual-core branch.

Weak-field normalization branch

If the weak-field coupling is independently normalized as

$$|\sigma| = \frac{16\pi G}{c_{\mathbb{E}}^2}, \quad (98)$$

then Eq. (94) yields the universal number

$$\boxed{\varepsilon_M = \frac{\sqrt{2}}{4} \simeq 0.353553}. \quad (99)$$

The same result follows from $Z\sigma^2 = 8\pi G$, which gives

$$Z = \frac{c_{\mathbb{E}}^4}{32\pi G}, \quad (100)$$

well inside the admissible domain (97). The implication is one-way: Eq. (98) fixes Eq. (99); the numerical residual fraction must not be used in reverse to select the weak-field normalization. Thus DSB-1 introduces no new residual-fraction parameter on this branch, while the branch itself remains conditional on the independent derivation or adoption of Eq. (98).

VI. Regular central limit

For the uniform residual-core reduction,

$$m(r) = \varepsilon_M M \left(\frac{r}{r_c} \right)^3, \quad 0 \leq r \leq r_c. \quad (101)$$

The constrained Newtonian acceleration is

$$g(r) = \frac{Gm(r)}{r^2} = \frac{G\varepsilon_M M}{r_c^3} r, \quad \boxed{g(0) = 0}. \quad (102)$$

and the potential has the finite expansion

$$\Psi(r) = \Psi(0) + \frac{G\varepsilon_M M}{2r_c^3} r^2. \quad (103)$$

This establishes regularity only within the uniform constrained reduction. It is not a proof of the complete nonlinear centre.

VII. The radial-pressure condition and the audited boundary flux

Introduce the static phase energy localized at the mechanical wall,

$$\mathcal{E}_\Sigma = \frac{\kappa_\chi}{2} (D_r \chi)^2 + W(\chi), \quad (104)$$

where W has the $\mathcal{A}$ and E branches as its two relevant minima. In the locally planar, degenerate-minimum limit, the first integral gives

$$\frac{\kappa_\chi}{2} (D_r \chi)^2 = W(\chi) - W_{\min}, \quad (105)$$

The effective energy per unit area carried by the mechanical wall is

$$\tau_\Sigma^{\text{eff}} = \int_{r_c - \Delta}^{r_c + \Delta} dr \left[\frac{\kappa_\chi}{2} (D_r \chi)^2 + W(\chi) - W_{\min} \right]. \quad (106)$$

In the thin-wall reduction the first integral converts this expression into

$$\boxed{\tau_\Sigma = \int_{\chi_{\mathcal{A}}}^{\chi_E} d\chi \sqrt{2\kappa_\chi [W(\chi) - W_{\min}]}.} \quad (107)$$

where Δ denotes the local wall thickness before the thin-wall limit. The tension is therefore a derived functional of κ_χ and W , not a parameter to be adjusted after the radius is known.

Define the total radial stress directly from the three-dimensional static action,

$$\Pi^r_r \equiv -\frac{2}{\sqrt{\gamma}} \frac{\delta S_{\text{static}}}{\delta \gamma_{rr}}. \quad (108)$$

The local thin-wall pressure condition at the inner saturation surface is

$$\boxed{[\Pi^r_r]_{r_c^-}^{r_c^+} = \frac{2\tau_\Sigma}{r_c}.} \quad (109)$$

Equation (78) supplies the integrated Born–Infeld derivative term

$$\begin{aligned} \mathcal{B}_{\text{BI}}(r_c) = -2u_\star \ell_\star^4 \oint_{r_c} dA n_i \Big[\nabla_j (F' \mathcal{R}^{ij}) \\ + \nabla^i (F' \mathcal{R}) \Big]. \end{aligned} \quad (110)$$

Its dimensional content follows directly from the definitions:

$$[u_\star] = \text{J m}^{-3}, \quad [\ell_\star^4] = \text{m}^4, \quad [\text{d}A] = \text{m}^2, \quad [\nabla \mathcal{R}] = \text{m}^{-3},$$

and therefore

$$[\mathcal{B}_{\text{BI}}] = \text{J}.$$

By contrast, $[\tau_\Sigma] = \text{J m}^{-2}$. Hence the local condition in Eq. (109) has pressure units,

$$\left[\frac{2\tau_\Sigma}{r_c} \right] = \text{J m}^{-3},$$

whereas

$$[8\pi r_c \tau_\Sigma] = \text{J m}^{-1} = \text{N}.$$

The equation formerly printed here,

$$\mathcal{B}_{\text{BI}}(r_c) + 8\pi r_c \tau_\Sigma = 0,$$

is therefore withdrawn: it adds an energy to a force, and it is not equivalent to Eq. (109).

AUDIT A-DSB-1-01 — NO SILENT REPAIR

A dimensionless spherical dilation of the pure surface energy $4\pi r_c^2 \tau_\Sigma$ would produce a wall contribution proportional to $8\pi r_c^2 \tau_\Sigma$, while a force-normalized balance would require an energy coefficient to be divided by a length. These are different variational normalizations and may not be interchanged. The combined finite-wall action must determine which boundary object is conjugate to which variation, regulate the divergence of $F'(X)$ as $X \rightarrow 1^-$, and derive the resulting dependence on r_c , the wall thickness and the constitutive parameters. No replacement global junction is asserted in this edition.

The need for a wall remains: the Born–Infeld density stays finite at $X = 1$, while its marginal response $F'(X)$ diverges. A naive continuation of the exterior profile to r_c therefore does not solve the boundary-value problem. What is withdrawn is not the need for phase matching, but the dimensionally inconsistent formula formerly used to represent it.

THE AUDITED VARIATIONAL LOCK OF DSB-1

Equations (65)–(97) freeze the optical–mechanical double scale, the conditional residual fraction and its admissible elastic domain. Equations (76)–(80) derive the exact bulk response of the nonlinear State $\mathcal{A}$ shell and prove its proportionality to M . On the independently normalized weak-field branch, Eqs. (98)–(100) additionally give $\varepsilon_M = \sqrt{2}/4$ without a new Black-Core parameter. OCH-1 states which sector carries each source role. The complete action must now derive a finite, dimensionally homogeneous wall–flux junction from the regulated phase profile, without a new fit, and must determine whether that junction preserves the same $r_c \propto M^{1/3}$ scale. If it selects another scale, requires a freely adjusted tension, gives E the same local scalar charge as residual matter, or requires a mass-dependent ε_M , DSB-1 is rejected rather than recalibrated.

Chapter 1

Fundamental energetic symmetry

1.1 The founding equation

Pillar II starts from the statement that the two great forms of cosmic energy are written with different natural variables while belonging to one structure:

ONE ENERGY, TWO PHYSICAL REGIMES

$$\mathcal{E}_{NC} = \mathcal{E}_M + \mathcal{E}_{\mathcal{A}} = mc^2 + N V_c .$$

Matter is concentrated energy; State $\mathcal{A}$ is energy distributed through the continuum.

For matter:

$$\mathcal{E}_M = mc^2 \tag{1.1}$$

For State $\mathcal{A}$:

$$\mathcal{E}_{\mathcal{A}} = N \cdot V_c \tag{1.2}$$

where:

- m denotes the mass in the matter branch;
- N denotes a density or energetic quantity proper to the State $\mathcal{A}$ regime;
- V_c denotes a volume or characteristic volume associated with this regime.

The two expressions are **distinct energetic writings** corresponding to **two regimes of a single reality**:

- **matter**: concentrated energy;
- **State $\mathcal{A}$** : diluted energy / transmitted through the continuum.

The common formalism must preserve the equivalence of the account while allowing each state to retain its own physical variables. This is the energetic symmetry from which the document takes its name.

1.2 Natural variables of each state

State	Natural variables	Equation
Matter	m, c	$\mathcal{E} = mc^2$
Electromagnetism	λ, ν, c	$\mathcal{E} = h\nu$
State $\mathcal{A}$	N, V_c	$\mathcal{E} = N \cdot V_c$
Cohesion E	field variables to be specified	calculation frontier

The table makes visible the RM statement: beyond the familiar variables of matter and radiation lies a regime in which energy is described through the **state and volume of the continuum**.

1.3 The electromagnetic transition toward State $\mathcal{A}$

The transition

$$\text{EM} \longrightarrow \mathcal{A}$$

is retained in RM as a **constitutive transition postulate**. Pillar II does not possess a microscopic action that derives its threshold, conversion rate, entropy production or efficiency. The later thermal analysis makes this distinction essential: the conditional Maria bound constrains a radiative loading, but does not by itself prove that reaching the bound selects $\text{EM} \rightarrow \mathcal{A}$.

The active working bridge is therefore written at its exact rank:

$$\begin{aligned} &\text{thermal/radiative loading} \longrightarrow \text{approach to a constitutive capacity} \\ &\longrightarrow \text{LF-09 channel closure} \longrightarrow \text{EM} \rightarrow \mathcal{A}. \end{aligned}$$

where the final arrow remains a postulated constitutive transition until a microscopic phase law derives it. No universal efficiency and no fitted thermal/redshift partition are inserted.

One representational question remains open: whether a transition between the electromagnetic description and the State $\mathcal{A}$ description can be written as a correspondence of natural variables of the type

$$(\lambda, \nu, c) \rightarrow (N, V_c).$$

This concerns the quantitative representation of the transition, not a derivation of its microscopic mechanism. It also leaves open the possibility of an intermediate state or a dynamic boundary. The transition must never be merged with the distinct transition $E \rightarrow m$ associated with Ignition in the Cosmic Core.

State $\mathcal{A}$ is not a co-product of this transition. Its existence is an independent RM postulate. The distinction is therefore between the ontological existence of State $\mathcal{A}$ and a possible change of its state under electromagnetic loading.

Transition memory

Rigidity closes the structure; the transition law closes the history.

What the conservative sectors of the White Paper govern is propagation; what this Pillar governs is change of state. On that ground, one refinement is recorded and one overreach is withdrawn.

Withdrawn: the proposition that every transition necessarily leaves a non-zero energetic residue of the preceding state (a “residual stock”). It is too strong and underived, and it is hereby removed from the candidate principles.

Recorded as an open hypothesis: transitions may inscribe a **memory** $\mathcal{M}$ in the structure of the resulting state — not a stock of the old state’s energy, but a trace in the organisation of the new one. This hypothesis has a demonstrated material realisation within the programme: in the conservative cosmological cycle (White Paper v6.8.5, chapter 18), mechanical energy is conserved to 10^{-10} across cycles, yet no two cycles are identical, because the thermonuclear fuel is consumed — a memory carried by matter, proven numerically. Its corollary is likewise established there: a closed energetic cycle does not entail historical repetition; conservation constrains the mechanics, not the history, of a universe with open time.

Whether a memory $\mathcal{M}$ exists at the level of the continuum itself — as a state variable of State $\mathcal{A}$ inscribed by the transitions of this Pillar — remains an open hypothesis. It will be admitted into the canonical structure only when endowed with a measurable consequence, in accordance with the programme’s acceptance rule.

1.4 State $\mathcal{A}$ as the condition of transmission

Without State $\mathcal{A}$, light can neither pass nor travel; gravitational manifestations cannot be transmitted; no structured gradient can propagate; no physical transmission is possible. **Without State $\mathcal{A}$, neither light nor transmitted gravitational manifestations can travel.**

The distinction is constitutional (PA-08, LF-01): gravity is fundamental; State $\mathcal{A}$ does not create the law of gravity; it permits its transmission and its observable manifestation. Likewise, electromagnetic radiation possesses its natural variables (λ, ν, c) , but its real propagation requires State $\mathcal{A}$: at the ideal limit represented by Silentium — which is not a physical continuum — radiation does not continue to propagate in its electromagnetic state; it changes state, $EM \rightarrow \mathcal{A}$.

1.5 The global causal ledger after the bulk audit

The current global architecture no longer compresses all cosmological energetics into a single chain. It separates three operations:

1. the homogeneous response of the already manifested bulk, described on the reduced branch by HBR-1 and conditionally extended by the thermal/curvature sector;

2. boundary production of new manifested State $\mathcal{A}$ volume, represented by Γ_{∂} and $g_{\partial} = \Gamma_{\partial}/(3V_{\mathcal{A}})$;
3. transfer to observables, including redshift, which requires its own propagation law and does not become an energy source merely by notation.

The total manifested-volume rate may therefore be written

$$H_{\mathcal{A}} = H + g_{\partial},$$

without identifying the two terms. HBR-1 closes the reduced deep-bulk rate under its declared ledger. It does not derive the boundary formation law, perturbations, or the observational expansion history.

Thermal-radiative loading is primary in the active stellar application. Redshift remains a genuine propagation observable and a possible secondary energetic contribution whose sign, receiver and quantitative share remain to be derived. The unit boundary law $\mathcal{U}_{\partial}\Gamma_{\partial} = \mathcal{P}_{\gamma,\partial}$ is retained only on its selected conditional branch; a full formation budget may contain layer storage, outgoing products and exchanged work.

The bulk rate is no longer missing. The boundary law, microscopic conversion and observational lift remain open.

1.6 Historical homogeneous branch: the HCR Exclusion attractor

STATUS — WITHDRAWN ACTIVE BRANCH / HISTORICAL RECORD

The equations in this section are preserved exactly because v4.6.0 declared HCR-1 provisional and required a later global action to reproduce or replace it. The later action-level homogeneous audit did not reproduce HCR-1. Under the non-repair rule, HCR-1/HCR-2 and their attractor consequences are no longer active closure equations. They remain a dated calculation showing the branch that was tested and removed.

Let $V_{\mathcal{A}}(T)$ denote the manifested State $\mathcal{A}$ volume and retain the current boundary/interior ledger

$$\dot{V}_{\mathcal{A}} = \int_{\Omega(T)} \Theta_{\mathcal{A}} dV + \Gamma_{\partial}. \quad (\text{HCR-0a})$$

For a homogeneous interior define

$$\Theta_{\mathcal{A}} = 3H, \quad H_{\mathcal{A}} \equiv \frac{1}{3} \frac{\dot{V}_{\mathcal{A}}}{V_{\mathcal{A}}}, \quad g_{\partial} \equiv \frac{\Gamma_{\partial}}{3V_{\mathcal{A}}}. \quad (\text{HCR-0b})$$

Then

$$H_{\mathcal{A}} = H + g_{\partial}. \quad (\text{HCR-0c})$$

This equation is kinematics only. It does not decide how much of the total manifested-volume growth is interior dilation and how much is formation of new boundary volume.

Introduce a dimensionless homogeneous deformation coordinate $e(T)$ for the already manifested interior,

$$\dot{e} = 3H, \quad (\text{HCR-0d})$$

with admissible domain

$$0 \leq e < e_{\text{cap}}. \quad (\text{HCR-0e})$$

The symbol e is not the historical swelling variable Γ . The notation Γ_{∂} remains reserved exclusively for boundary production.

1.6.1 HCR-1 — provisional rate relation

The homogeneous constitutive branch is defined by

$$r \equiv \frac{H}{H_{\mathcal{A}}} = \frac{p_{\mathcal{A}} - \varepsilon_z(e)}{\varepsilon'_z(e)}. \quad (\text{HCR-1})$$

Here $p_{\mathcal{A}}$ is written in the Pillar-II thermodynamic convention inherited by PS-0e: positive $p_{\mathcal{A}}$ is a positive pressure contribution. No sign flip to a separate tensile variable is made.

Equation (HCR-1) is dimensionally consistent when e is dimensionless and $p_{\mathcal{A}}$ and ε_z are energy densities. It is not yet a theorem of the full action. The future global action must derive this precise relation or replace the branch.

1.6.2 The hyperelastic Exclusion spring

The minimal barrier tested in this branch is

$$\varepsilon_z(e) = \kappa \frac{e}{e_{\text{cap}} - e}, \quad \kappa > 0. \quad (\text{HCR-2})$$

With

$$\Delta \equiv e_{\text{cap}} - e > 0, \quad (1.1)$$

one has

$$\varepsilon'_z(e) = \kappa \frac{e_{\text{cap}}}{\Delta^2} > 0. \quad (\text{HCR-2a})$$

Thus the stored load diverges as Δ^{-1} while its tangent stiffness diverges as Δ^{-2} . The barrier is a candidate quantitative realisation of LF-09; LF-09 itself does not constitutionally select this rational function.

Substitution into HCR-1 gives

$$r = \frac{\Delta^2 p_{\mathcal{A}}}{\kappa e_{\text{cap}}} - \frac{e \Delta}{e_{\text{cap}}}. \quad (\text{HCR-2b})$$

Therefore $r \rightarrow 0$ if

$$p_{\mathcal{E}} = o(\Delta^{-2}). \quad (\text{HCR-2c})$$

This limit alone does not fix the sign of r . On the regular finite-load branch, or more generally whenever $p_{\mathcal{E}} < \varepsilon_z$ sufficiently near the capacity, $r < 0$ and the capacity surface is dynamically repulsive. A pressure that diverges as fast as or faster than the tangent stiffness belongs to a different constitutive branch and cannot be inserted merely to defeat the barrier.

1.6.3 The sub-capacity equilibrium

The zero of the deformation rate is determined before the capacity:

$$r = 0 \quad \Longleftrightarrow \quad \boxed{p_{\mathcal{E}} = \varepsilon_z(e_*)}. \quad (\text{HCR-3a})$$

For a finite positive pressure treated as fixed over the mechanical relaxation interval,

$$\boxed{e_* = e_{\text{cap}} \frac{p_{\mathcal{E}}}{\kappa + p_{\mathcal{E}}} < e_{\text{cap}}}. \quad (\text{HCR-3b})$$

A generic linear spring can also have a force-balance point. The specifically LF-09 content of HCR-2 is that the admissible capacity is protected for every finite positive load: the equilibrium cannot be pushed beyond e_{cap} by merely increasing a finite $p_{\mathcal{E}}$.

THE EXCLUSION-ATTRACTOR STATEMENT

$$\begin{aligned} & \text{finite positive load} \implies e_* < e_{\text{cap}} \\ \implies & \text{the admissible branch organises before the capacity is crossed} \end{aligned}$$

This is a theorem of HCR-1 plus HCR-2, not yet an unconditional theorem of RM.

1.6.4 Stability and relaxation

Allow $p_{\mathcal{E}} = p_{\mathcal{E}}(e)$. Differentiating HCR-1 and evaluating at $p_{\mathcal{E}} = \varepsilon_z$ gives

$$\boxed{r'(e_*) = \frac{p'_{\mathcal{E}}(e_*)}{\varepsilon'_z(e_*)} - 1}. \quad (\text{HCR-4a})$$

Since $\dot{e} = 3H_{\mathcal{E}}r$, an expanding branch $H_{\mathcal{E}} > 0$ is locally attractive when

$$\boxed{p'_{\mathcal{E}}(e_*) < \varepsilon'_z(e_*)}. \quad (\text{HCR-4b})$$

For a locally constant load, $p'_{\mathcal{E}} \simeq 0$ and

$$\delta \dot{e} \simeq -3H_{\mathcal{E}} \delta e, \quad \boxed{\tau_{\text{relax}} \simeq (3H_{\mathcal{E}})^{-1}}. \quad (\text{HCR-4c})$$

1.6.5 A moving attractor has a finite dynamical thickness

The State- $\mathcal{A}$ pressure need not be constant. If it evolves slowly enough that the linear relaxation analysis remains meaningful,

$$e_*(T) = e_{\text{cap}} \frac{p_{\mathcal{A}}(T)}{\kappa + p_{\mathcal{A}}(T)}, \quad (1.2)$$

so

$$\dot{e}_* = e_{\text{cap}} \frac{\kappa \dot{p}_{\mathcal{A}}}{(\kappa + p_{\mathcal{A}})^2}. \quad (\text{HCR-5a})$$

The quasistatic tracking branch therefore carries

$$r_{\text{track}} \simeq \frac{\dot{e}_*}{3H_{\mathcal{A}}} = \frac{e_{\text{cap}}}{3} \frac{\kappa p_{\mathcal{A}}}{(\kappa + p_{\mathcal{A}})^2} \frac{\dot{p}_{\mathcal{A}}/p_{\mathcal{A}}}{H_{\mathcal{A}}}. \quad (\text{HCR-5b})$$

For positive $p_{\mathcal{A}}$ and κ ,

$$\frac{\kappa p_{\mathcal{A}}}{(\kappa + p_{\mathcal{A}})^2} \leq \frac{1}{4}, \quad (1.3)$$

which gives the useful bound

$$|r_{\text{track}}| \lesssim \frac{e_{\text{cap}}}{12} \left| \frac{\dot{p}_{\mathcal{A}}/p_{\mathcal{A}}}{H_{\mathcal{A}}} \right|. \quad (\text{HCR-5c})$$

Thus exact $r = 0$ is the stationary/adiabatic limit. A time-dependent load produces a calculable lag rather than destroying the attractor.

1.6.6 Boundary production at exact equilibrium

At $r = 0$, one has $H = 0$. The kinematic ledger then closes immediately:

$$H_{\mathcal{A}} = g_{\partial} = \frac{\Gamma_{\partial}}{3V_{\mathcal{A}}}. \quad (\text{HCR-6a})$$

Using the currently accepted minimal boundary branch,

$$\Gamma_{\partial} = \mathcal{K}_{\text{cryst}} \mathcal{P}_{\gamma, \partial}, \quad \mathcal{K}_{\text{cryst}} = \frac{\ell_{\star}^2}{P}, \quad (1.4)$$

there follows

$$H_{\mathcal{A}}|_{r=0} = \frac{\mathcal{K}_{\text{cryst}} \mathcal{P}_{\gamma, \partial}}{3V_{\mathcal{A}}}. \quad (\text{HCR-6b})$$

This statement does not derive an observed Hubble history. It says something narrower and exact within the stated ledgers: at exact constitutive equilibrium, all manifested-volume growth is boundary production and none is additional homogeneous dilation of the already manifested interior.

1.6.7 Sign ledger and the global-action debt

The stored reservoir ε_z is kept positive on HCR–2. No separate compensator $\mathcal{C}_G$ is inserted here to make an un-derived homogeneous Hamiltonian balance close. That earlier diagnostic route is not part of the present proof. The correct global Hamiltonian constraint and its sign ledger must be obtained from the future global action and must then reproduce or replace HCR–1.

Accordingly, the present result has a deliberately asymmetric status:

- the algebraic existence and stability statements above are derived from HCR–1/HCR–2;
- HCR–1 itself is a constitutive postulate of the homogeneous candidate branch;
- the positive sign and microscopic origin of $p_{\mathcal{E}}$ remain to be derived from the homogeneous State- $\mathcal{E}$ stress–energy tensor;
- κ and e_{cap} may not be calibrated from H_0 , q_0 , a Friedmann history or the Conicus3D prior;
- if the global action produces extra terms, a different sign, or a different rate relation, this branch is replaced rather than repaired.

LF–09 — v4.7.0 READING

LF–09 survives; the HCR realisation does not. The Exclusion Interaction states the constitutive refusal of indefinite continuation through a channel as its admissible capacity is approached. HCR–1/HCR–2 are retained only as one historical attempted realisation. The active homogeneous calculation is the constraint-based HBR branch introduced next.

1.7 Current homogeneous closure: HBR–1 and the thermal extension

STATUS — ACTIVE HOMOGENEOUS AUTHORITY

The narrow authority for the homogeneous zero mode is now *The Uncollapsing Universe: Constitutive Exclusion and Homogeneous Bulk Closure in Relativity of Matter*, v4.0.0. HCR–1 is historical. The reduced $U = 0$ theorem and its thermal extension below are kept distinct.

For the reduced homogeneous branch, the surviving scalar constraints are

$$P^{(3)}R = \rho_m + u_\gamma + \varepsilon_z + u_0, \quad (\text{GHC–1})$$

$$\rho_m + 2u_\gamma - 2u_0 = 2\varepsilon_z + 3\varepsilon'_z. \quad (\text{GEC–1})$$

Define

$$B(e) = 2\varepsilon_z(e) + 3\varepsilon'_z(e).$$

Under

$$\dot{\rho}_m = -3H\rho_m - s, \quad \dot{u}_\gamma = -4Hu_\gamma + s, \quad \dot{e} = 3H,$$

preservation of GEC-1 gives

$$\boxed{H [3B'(e) + 3\rho_m + 8u_\gamma] = s.} \quad (\text{HBR-1})$$

GHC-1 is then preserved identically. No additional homogeneous rate equation, Friedmann law, H_0 or fitted response coefficient is required.

The active stellar energetic interpretation is not obtained by renaming ε_z . Introduce instead the thermal stock $\varepsilon_{\mathcal{E}}^{\text{th}}(e)$ and the curvature-Exclusion energy

$$X = \frac{\ell_\star^4 ({}^{(3)}R)^2}{3}, \quad 0 \leq X < 1, \quad (1.5)$$

$$U(X) = u_\star F(X), \quad F(X) = 1 - \sqrt{1 - X} - \frac{X}{2}. \quad (1.6)$$

The homogeneous thermal constraints are

$$P({}^{(3)}R) = \rho_m + u_\gamma + \varepsilon_{\mathcal{E}}^{\text{th}} + u_0 + U, \quad (1.7)$$

$$\rho_m + 2u_\gamma - 2u_0 = 2\varepsilon_{\mathcal{E}}^{\text{th}} + 3(\varepsilon_{\mathcal{E}}^{\text{th}})' + 2U - 4XU'. \quad (1.8)$$

With

$$B_{\text{th}} = 2\varepsilon_{\mathcal{E}}^{\text{th}} + 3(\varepsilon_{\mathcal{E}}^{\text{th}})', \quad \dot{X} = -4HX,$$

constraint preservation gives

$$\boxed{H [3B'_{\text{th}} + 8XU' + 16X^2U'' + 3\rho_m + 8u_\gamma] = s.} \quad (\text{HBR-TH})$$

For $U = 0$ this reduces to HBR-1. The thermal equation is therefore an extension under additional assumptions, not a retroactive reinterpretation of the reduced theorem.

For equilibrium radiation,

$$u_\gamma = a_R \Theta^4, \quad a_R = \frac{\pi^2 k_B^4}{15 \hbar^3 c_{\mathcal{E}}^3}.$$

Under UC-1,

$$\boxed{u_{\mathcal{E}}^{\text{cap}} = \frac{P}{\ell_\star^2}} \quad (\text{postulate}),$$

the Maria reduction gives

$$\boxed{u_\gamma \leq \frac{5P}{4\ell_\star^2} - u_0}$$

and hence

$$\boxed{\Theta \leq \left[\frac{1}{a_R} \left(\frac{5P}{4\ell_\star^2} - u_0 \right) \right]^{1/4}}$$

when the argument is non-negative. No universal temperature in kelvins follows until $\ell_\star$ is independently fixed. The equality need not be attained.

THERMAL-TO-STATE BRIDGE

The further sequence “approach to the Maria bound $\rightarrow$ LF-09 channel closure $\rightarrow EM \rightarrow \mathcal{A}$ ” is a working constitutive bridge, not a microscopic theorem. Thermal-radiative loading is primary in the active stellar interpretation. Redshift remains physical and may contribute secondarily, but its energetic sign, receiver and quantitative share are not fixed by this calculation.

The global kinematic separation remains

$$H_{\mathcal{A}} = H + g_{\partial}, \quad g_{\partial} = \frac{\Gamma_{\partial}}{3V_{\mathcal{A}}}.$$

Boundary production, formation cost, perturbations and transfer to observables are separate sectors. In particular, $P/\ell_{\star}^2$ is the natural volumetric scale, while $\mathcal{U}_{\partial} = P/\ell_{\star}^2$ is only a conditional unit boundary branch. It is not by itself the residual energy density deposited in new volume.

1.8 $\mathcal{A}$: The Missing State of the Universe and the Exclusion Interaction

Matter is the concentrated state, electromagnetic radiation is the propagating state, and E is the state of maximal cohesion. RM names $\mathcal{A}$ as the missing distributed state: the physical body that carries propagation, constitutive work and energetic relations between material structures.

PS-0 — HISTORICAL GENEALOGY ONLY

The PS-0 equations are retained below as a dated radiative ansatz. They no longer define the active stellar or homogeneous energy ledger. In particular, the old continuous redshift-work channel is not used as the primary constitutive loading of State $\mathcal{A}$; HCR-1/HCR-2 are likewise historical. LF-09 survives independently as the Exclusion Interaction.

This statement is represented at the present stage by a deliberately minimal native RM closure. It imports neither the Einstein field equation nor a Schwarzschild solution. It uses only the photon energy E_{γ} , its momentum p_{γ} , the saturation speed $c_{\mathcal{A}}$, the production-driven volume rate of the manifested domain, and the two opposed energetic accounts.

1.8.1 PS-0 — historical joint constitutive ansatz

Introduce one positive dimensionless coefficient,

$$\kappa_S > 0, \tag{PS-0}$$

called the **saturation coefficient**. The electromagnetic branch is defined by the saturation surface

$$\boxed{E_{\gamma}^2 - \kappa_S^2 c_{\mathcal{A}}^2 p_{\gamma}^2 = 0} \tag{PS-0a}$$

and therefore

$$E_\gamma = \kappa_S c_\mathcal{A} p_\gamma, \quad v_\gamma = \frac{\partial E_\gamma}{\partial p_\gamma} = \kappa_S c_\mathcal{A}. \quad (\text{PS-0b})$$

Let $V_\mathcal{A}(T)$ denote the physical volume occupied by State $\mathcal{A}$. Expansion is attributed to boundary production, not to the stretching of an existing body. Its macroscopic rate is

$$\boxed{H_\mathcal{A} \equiv \frac{1}{3} \frac{\dot{V}_\mathcal{A}}{V_\mathcal{A}}}. \quad (\text{PS-0c})$$

The same coefficient is then assigned to the continuous redshift-work channel:

$$\dot{u}_\gamma + 3H_\mathcal{A} u_\gamma = -\kappa_S H_\mathcal{A} u_\gamma, \quad (\text{PS-0d})$$

$$\dot{u}_\mathcal{A} + 3H_\mathcal{A} (u_\mathcal{A} + p_\mathcal{A}) = +\kappa_S H_\mathcal{A} u_\gamma. \quad (\text{PS-0e})$$

Thus

$$\boxed{Q_{\text{EM} \rightarrow \mathcal{A}}^{\text{sat}} = \kappa_S H_\mathcal{A} u_\gamma} \quad (\text{PS-0f})$$

is lost by the electromagnetic account and received with the opposite sign by the continuum account. Addition of (PS-0d) and (PS-0e) gives

$$\dot{u}_{\text{tot}} + 3H_\mathcal{A} (u_{\text{tot}} + p_{\text{tot}}) = 0. \quad (\text{PS-0g})$$

The ansatz therefore closes the homogeneous energy ledger exactly.

1.8.2 Single calibration and consequences

For an individual photon, (PS-0d) separates volumetric dilution from the change of photon energy and yields

$$\frac{d \ln E_\gamma}{d \ln a} = -\kappa_S. \quad (\text{PS-0h})$$

Consequently,

$$u_\gamma \propto a^{-(3+\kappa_S)}, \quad 1+z = \left(\frac{a_0}{a_{\text{em}}} \right)^{\kappa_S}. \quad (\text{PS-0i})$$

The observed achromatic cosmological-redshift exponent fixes the single calibration

$$\boxed{\kappa_S = 1.} \quad (\text{PS-0j})$$

Once fixed, this value may not be readjusted in the propagation or energetic channels. PS-0 then gives simultaneously

$$\boxed{v_\gamma = c_\mathcal{A}, \quad Q_{\text{EM} \rightarrow \mathcal{A}}^{\text{sat}} = H_\mathcal{A} u_\gamma.} \quad (\text{PS-0k})$$

The equality $v_\gamma = c_\mathcal{A}$ becomes an independent numerical prediction only after $c_\mathcal{A}$ has been normalized from a physical datum that does not already use the measured speed of light. Until that normalization is achieved, it is a constitutive consequence of PS-0, not an independent determination of c .

1.8.3 Continuous work and boundary conversion are distinct

PS-0 describes the continuous transfer associated with redshift along propagation. It must not be confused with the state conversion that occurs at the physical boundary of the $\mathcal{A}\mathcal{E}$ domain:

Channel	Physical role	Coefficient
$Q_{\text{EM} \rightarrow \mathcal{A}\mathcal{E}}^{\text{sat}}$	continuous constitutive transfer during propagation	κ_{S}
$Q_{\text{EM} \rightarrow \mathcal{A}\mathcal{E}}^{\Sigma}$	boundary conversion and extension of the $\mathcal{A}\mathcal{E}$ domain	β

The coefficients are not interchangeable. In particular, the effective source $\beta S_{\star}$ used in the numerical checkpoint below compresses production, transport and conversion into one homogeneous source. It must not be added mechanically to (PS-0f) without first constructing the causal transport and boundary ledger; otherwise the same photon energy could be counted twice.

1.8.4 Immediate consistency tests

The ansatz has four direct limiting properties:

- (i) if $H_{\mathcal{A}\mathcal{E}} = 0$, the continuous exchange vanishes;
- (ii) if $H_{\mathcal{A}\mathcal{E}} < 0$, the sign reverses and the contracting continuum returns energy to the electromagnetic branch;
- (iii) a constant κ_{S} makes the transfer achromatic and independent of the luminosity of an individual source;
- (iv) the two sector balances add to an exact total conservation equation.

A frequency-dependent redshift exponent, a luminosity-dependent propagation law, unequal values of κ_{S} inferred from propagation and redshift, or failure of the opposed energy ledger would falsify PS-0.

STATUS OF PS-0 AFTER v4.7.0

PS-0 remains an archived model with its own internal falsification conditions. Its common coefficient κ_{S} is no longer an active calibration linking the current thermal and redshift sectors. Redshift is retained as a propagation observable and possible secondary energetic channel; no current equation fixes a universal thermal/redshift partition.

1.8.5 Historical radiative-expansion checkpoint

HISTORICAL OBSERVATIONAL CHECKPOINT — REPRODUCIBLE EFFECTIVE MODEL. The causal chain above has been implemented in a reproducible homogeneous

model in order to test whether a radiatively sourced State $\mathcal{E}$ sector can generate a viable late-time expansion history. The final self-consistent candidate studied in the accompanying notebook is

$$Q_{\text{EM} \rightarrow \mathcal{E}}(t) = \beta S_\star[z(t)], \quad (1.3a)$$

$$\dot{X}_{\mathcal{E}}(t) = Q_{\text{EM} \rightarrow \mathcal{E}}(t) - 3H(t)(1 + w_{\mathcal{E},\text{eff}})X_{\mathcal{E}}(t), \quad (1.3b)$$

$$H^2(t) = C_m a^{-3}(t) + \alpha_X X_{\mathcal{E}}(t). \quad (1.3c)$$

Here $S_\star$ is the stellar-radiation source reconstructed on the evolving RM background; $X_{\mathcal{E}}$ is a normalized accumulated state variable, not yet the physical density $\rho_{\mathcal{E}}$; $w_{\mathcal{E},\text{eff}}$ is an effective work coefficient; and α_X absorbs the present normalization of the source. Consequently, the numerical value of α_X is not yet a fundamental microscopic coupling.

The rigid-delay hypothesis was tested first. When confronted with Pantheon+ supernovae, the likelihood drove the imposed delay toward zero. A fixed multi-gigayear delay is therefore **not retained** as the working mechanism. In the self-consistent source model, a one-parameter fit of the effective work coefficient gives the checkpoint

$$\boxed{w_{\mathcal{E},\text{eff}} = -0.683953, \quad q_0 = -0.405656, \quad z_t = 0.834140} \quad (1.4)$$

with

$$\Delta\chi^2_{\text{Pantheon+}} = \chi^2_{\text{RM}} - \chi^2_{\Lambda\text{CDM}} \simeq +0.244 \quad (\text{numerically stable at the } 10^{-3} \text{ level}) \quad (1.5)$$

relative to the optimized flat- Λ CDM comparison used in the notebook. In the same normalization, the present energy accounting gives approximately 48.849% retained in the internal State $\mathcal{E}$ sector and 51.151% transferred as cumulative constitutive work, with a numerical closure residual bounded by $\varepsilon_{\text{closure}} < 4 \times 10^{-6}$. Independent evaluations of this residual differ at the 10^{-6} level; neither the smallest nor the most favourable is adopted. The definitive value must come from a step-halving convergence test reporting the observed order of convergence and the extrapolated limit, not merely the smallest residual obtained. A diagonal cosmic-chronometer diagnostic gives a difference of order $\Delta\chi^2 \simeq +0.95$ relative to the comparison model; the complete systematic covariance has not yet been included.

These values establish *numerical viability of a candidate effective history*, not the truth of the constitutive law. In particular:

- $w_{\mathcal{E},\text{eff}}$ is fitted and must be derived from the continuum stress-energy tensor;
- $\beta = 0.9865$ encodes the tested 1.35% post-contact electromagnetic residue and remains a constitutive hypothesis;
- the source amplitude is degenerate with α_X ;
- the calculation starts after the earliest radiation era and does not yet include perturbations, BAO, CMB anisotropies, or growth of structure.

1.8.6 Dictionary of quantities that must remain distinct

The numerical checkpoint does not identify the normalized variable $X_{\mathcal{E}}$ with the densities used elsewhere in Pillar II. Until the action, its stress–energy tensor, and the source normalization close the chain, the following separation is mandatory:

$$\boxed{\rho_E^{\max} \neq N_{\text{sat}} \neq \rho_{\mathcal{E},0} \neq X_{\mathcal{E}}^{\text{notebook}}} \quad (1.6)$$

where $\rho_E^{\max}$ is the provisional terminal cohesion density, N_{sat} is the Hamiltonian energy density $\mathcal{H}_{\text{sat}}$ of a specified saturated background, $\rho_{\mathcal{E},0}$ is the present physical continuum density, and $X_{\mathcal{E}}^{\text{notebook}}$ is a normalized effective variable. Any equality among them requires an independent derivation.

Chapter 2

The fundamental energetic state E

2.1 Definition and properties

The symbol E denotes the state of **maximal energetic cohesion** of the framework. The two energetic branches converge toward it:

$$m \rightarrow E + \varepsilon_M \quad (2.1)$$

$$\mathcal{A} \rightarrow E + \varepsilon \quad (2.2)$$

where ε_M and ε denote the residual exceptions associated with each branch, and m is the canonical symbol of the matter state in transition chains (the symbol M is reserved below for measured masses).

The three canonical critical transitions of the framework (PA-10) are therefore written explicitly:

$$m \longrightarrow E, \quad \mathcal{A} \longrightarrow E, \quad E \xrightarrow[\text{Cosmic Core}]{\text{Ignition}} m.$$

$m \rightarrow E$ and $\mathcal{A} \rightarrow E$ are transitions *toward maximal cohesion*; $E \rightarrow m$ is a *distinct* transition, tied to the Ignition within the Cosmic Core, and must never be merged with $\text{EM} \rightarrow \mathcal{A}$. No real transition totally erases memory, information or asymmetry (LF-05).

Thus, matter and State $\mathcal{A}$ are not conceived as two independent ultimate realities, but as two distinct trajectories toward a single state of cohesion.

Properties attributed to E

- **Incompressibility:** E is not assumed to be infinitely compressible. The model replaces the perfect singularity with a limit of cohesion.
- **Maximal cohesion:** E represents a limiting state of cohesive energetic concentration.
- **Minimal exception:** in accordance with the transition hierarchy adopted in v3.5.3 (conservation $\rightarrow$ saturation $\rightarrow$ transition $\rightarrow$ residual trace), no state is absolutely perfect. The exception ε is no longer a driving causal law but a residual memory or trace of the transition process.

Central conceptual point

Maximal cohesion does not necessarily imply minimal volume.

This sentence plays a fundamental role in Pillar II. The Cosmic Core is not conceived as a point of zero volume, but as a state of extreme cohesion still endowed with a finite extension. It is one of the clearest departures between the Foundation/TRD framework and the classical reading of the singularity.

In other words, Pillar II does not replace a singularity with a mere “big black hole”. It proposes another reading of the final state: not an object of minimal size, but a **state of maximal cohesion compatible with an extended structure**.

2.2 Density limit of E — calculation

Two levels must be strictly distinguished here. The **Absolute Principle** (PA-06, LF-08): E possesses a fundamental, impassable density limit — this is constitutional and does not depend on any numerical estimate. And the **numerical estimate** of this document: a *provisional estimate of the maximal energy density associated with E under the current working assumptions*, dependent on the R_{NC} hypothesis, on the total energy retained, on the assumed volume of the Cosmic Core, and on the cosmological estimates used.

If the Cosmic Core constitutes the final cohesion state of the cycle, then an estimate of the maximal energy density associated with E can be obtained by relating the total energy of the system to the volume of the Core:

$$\rho_E^{\max} \approx \frac{\mathcal{E}_{\text{total}}}{V_{NC}} = \frac{\mathcal{E}_M + \mathcal{E}_E}{\frac{4}{3}\pi R_{NC}^3} \quad (2.3)$$

Working hypotheses

- diameter of the Cosmic Core:

$$D_{NC} = 500\,000 \text{ ly} \quad \Rightarrow \quad R_{NC} = 250\,000 \text{ ly} \approx 2.365 \times 10^{21} \text{ m}$$

- observable baryonic mass:

$$M_{\text{baryons}} \approx 10^{53} \text{ kg}$$

- current dark-energy density:

$$\rho_{\Lambda} \approx 6 \times 10^{-10} \text{ J/m}^3$$

These hypotheses do not all have the same status. The observable baryonic mass and the order of magnitude of ρ_{Λ} refer to known cosmological estimates. The diameter of the Cosmic Core, by contrast, remains at this stage a **working hypothesis** and not a quantity derived from the model.

2.2.1 Matter branch

The energy of the matter branch is estimated by:

$$\mathcal{E}_M = M_{\text{baryons}} c^2 \approx 10^{53} \times (3 \times 10^8)^2 \approx 9.0 \times 10^{69} \text{ J} \quad (2.4)$$

2.2.2 State $\mathcal{A}E$ branch

The volume of the observable Universe is approximated by:

$$V_{\text{obs}} = \frac{4}{3} \pi R_{\text{obs}}^3 \approx 3.45 \times 10^{80} \text{ m}^3 \quad (2.5)$$

with $R_{\text{obs}} \approx 46 \times 10^9$ light-years.

The energy associated with the State $\mathcal{A}E$ branch is then:

$$\mathcal{E}_{\mathcal{A}E} = \rho_{\Lambda} V_{\text{obs}} \approx 6 \times 10^{-10} \times 3.45 \times 10^{80} \approx 2.07 \times 10^{71} \text{ J} \quad (2.6)$$

Ratio of the two branches

$$\frac{\mathcal{E}_{\mathcal{A}E}}{\mathcal{E}_M} \approx 23 \quad (2.7)$$

The State $\mathcal{A}E$ branch thus dominates the matter branch today by a factor of about **23**. In this framework, the final Cosmic Core would not be mainly a residue of reconcentrated matter, but **State $\mathcal{A}E$ reconverted into cohesion**.

2.2.3 Maximal cohesion energy density

The total energy is:

$$\mathcal{E}_{\text{total}} = \mathcal{E}_M + \mathcal{E}_{\mathcal{A}E} \approx 9.0 \times 10^{69} + 2.07 \times 10^{71} \approx 2.16 \times 10^{71} \text{ J} \quad (2.8)$$

The volume of the Core is:

$$V_{NC} = \frac{4}{3} \pi R_{NC}^3 \approx \frac{4}{3} \pi (2.365 \times 10^{21})^3 \approx 5.54 \times 10^{64} \text{ m}^3 \quad (2.9)$$

Hence:

$$\rho_E^{\text{max}} \approx \frac{2.16 \times 10^{71}}{5.54 \times 10^{64}} \approx 3.9 \times 10^6 \text{ J/m}^3 \quad (2.10)$$

HISTORICAL WORKING ESTIMATE

Equation (2.10) preserves the original 250 000 ly radius hypothesis and its associated top-hat mean density. It is not the later PF-e-R1 anchoring calculation. The integrity-audited quadratic branch is developed in §3.7 and gives $u_{E,\max}^{\text{anc}} = 3.3778 \times 10^6 \text{ J/m}^3$ together with the conditional specification-A scale $R_{NC}^{\text{anc}}(A) = 262\,211.6 \text{ ly}$. Both are retained because they answer different questions: historical construction versus constitutive anchoring.

2.2.4 Comparisons

State	Energy density	Ratio with $\rho_E^{\max}$
Current dark energy ρ_Λ	$6 \times 10^{-10} \text{ J/m}^3$	$\times 6.5 \times 10^{15}$
Cosmic Core $\rho_E^{\max}$	$3.9 \times 10^6 \text{ J/m}^3$	1
Nuclear density	$\sim 2 \times 10^{34} \text{ J/m}^3$	$\div 5 \times 10^{27}$
Planck density	$\sim 10^{113} \text{ J/m}^3$	$\div 2.6 \times 10^{106}$

The Cosmic Core is therefore about 6.5×10^{15} times denser than current dark energy, while remaining immensely less dense than an atomic nucleus or a Planck density. In the Foundation/TRD framework, it is therefore not a singularity, but an **extended state of cohesion with a finite maximal energy density**.

Chapter 3

The Cosmic Core

3.1 Definition and equations

The **Cosmic Core** is the final state of maximal cohesion toward which the two energetic branches of the model converge. It must not be understood as a point, but as an **extended energetic structure** representing the convergence limit of matter and State $\mathcal{A}$ toward E .

Its energetic formulation reads:

$$\mathcal{E}_{NC} = \mathcal{E}_M + \mathcal{E}_{\mathcal{A}} = mc^2 + N \cdot V_c \quad (3.1)$$

$$NC \rightarrow E + \varepsilon_M + \varepsilon \quad (3.2)$$

At this stage, its volume can be approximated as the sum of the final Schwarzschild volumes associated with the Black Cores of the scenario:

$$V_{NC} = \sum_i \frac{4}{3} \pi R_{BH,i}^3, \quad R_{BH,i} = \frac{2GM_i}{c^2} \quad (3.3)$$

External-framework bridge — the Schwarzschild radius $R_{BH,i} = 2GM_i/c^2$ is retained as a comparative or observational relation, not derived here from the internal dynamics of Foundation/TRD. The sum of Schwarzschild volumes is an **auxiliary construction** and a **conditional estimate**: it does not constitute a definitive internal derivation of the volume of the Core. It serves as a **construction scheme**: the Core is treated as a global final state resulting from the convergence of the ultimate compact structures.

3.2 Comparison with general relativity

The conceptual difference between classical general relativity and the Foundation/TRD framework can be summarized as follows:

Classical general relativity	Foundation/TRD
initial volume $\rightarrow 0$	initial volume $\rightarrow V_{NC} \neq 0$
density $\rightarrow \infty$	finite maximal density $\rho_E^{\max}$
perfect singularity	extended maximal cohesion
Big Bang as a point	Cosmic Core as a structure

Classical general relativity cancels the initial volume. The Foundation/TRD framework, by contrast, assigns it a **finite radius**. This comparison is non-normative: general relativity does not determine or validate the Absolute Principles; it is used here as an external comparative frame only. Between the two lies, perhaps, the quantity that appears today as the cosmological constant.

3.3 Size estimates

The diameter of the Cosmic Core is not yet derived from a single principle. At this stage it results from a **progressive construction** based on successive bounds.

Cosmic Core diameter	Hypothesis	Status
120 000 ly	visible matter only	lower bound
240 000 ly	$\times 2$ correction for unseen matter	intermediate estimate
500 000 ly	adopted working hypothesis	current reference value
$> 10^6$ ly	with full State $\mathcal{A}E$ branch	possible upper estimate

The diameter of 500 000 light-years is therefore not presented as a demonstrated result, but as a **high working hypothesis**, consistent with the idea that the minimal calculation based on visible matter alone necessarily underestimates the final size of the Core.

3.4 Physical meaning of this progression

The estimate at 120 000 light-years comes from a calculation based on the matter branch alone, already accounted for. It does not take into account:

- dark matter and unobserved matter;
- galaxies beyond the horizon;
- the State $\mathcal{A}E \rightarrow E$ branch;
- the energetic legacy of past radiation.

The value 500 000 ly must therefore be understood as a **working hypothesis consistent with a progressive correction of the lower bound**, and not as an arbitrary number.

The Cosmic Core is not conceived as the smallest possible object. It is conceived as the state of maximal possible cohesion.

This sentence must be taken seriously: it forbids reading the Core as a mere compactified residue of standard cosmology. The Core is a hypothesis of global energetic architecture, not merely an extrapolation of the terminal black hole.

3.5 Zero global angular momentum

A structural property is added to the definition of the Cosmic Core. A state of maximal cohesion is a state from which the large global asymmetries have disappeared; overall rotation is one of them. We therefore posit, as a **structural condition** of maximal cohesion, the vanishing of the global angular momentum of the final state:

$$J_{\text{final}} = 0 \quad (3.4)$$

This is not a claim about the dynamics of a particular pair of merging Black Cores, but a condition on the final state itself. A residual net angular momentum would define a privileged global axis; the final state would then no longer possess the symmetry properties required to serve as the origin of a new cosmological cycle. Several dynamical histories can satisfy this condition: the vanishing is the **constraint**, whereas a compensation of successive angular momenta,

$$\sum_i J_i \rightarrow 0 \quad (3.5)$$

is only one possible **mechanism**.

The net global angular momentum may vanish through compensation, while local angular momenta, residual asymmetries, internal dynamics and memory remain non-zero. $J_{\text{final}} = 0$ therefore does not mean perfect immobility, the disappearance of all local rotation, perfect symmetry, the erasure of asymmetries, or the disappearance of dynamical memory — Such persistence belongs to the open transition-memory hypothesis and to the non-repetition clause LF-05; it is not elevated here to a universal law. The Cosmic Core thus appears not only as a maximum of energetic cohesion ($\rho_E^{\text{max}} \approx 3.9 \times 10^6 \text{ J/m}^3$, provisional estimate), but also as a **minimum of residual net structure**. In this way a third terminal condition is added to the model, alongside energy conservation and the possible persistence of historical difference: the final vanishing of the global angular momentum.

3.6 Memory of the Cosmic Core

The Cosmic Core preserves, in cohesive form, structural information about the history that produced it. Three levels are distinguished, articulated but distinct.

Cohesion memory (interpretation). As the energetic content of the Universe converges toward E , the accumulated energetic legacy — radiation, the State $\mathcal{A}E$ continuum, formed structures, Black Cores — is preserved in the cohesive state. Since the State $\mathcal{A}E$ branch dominates by a factor of 23 (§2.2), this memory is above all that of the reconverted continuum, and not that of matter.

Memory of matter (interpretation). The history of matter is a *component* of cohesion memory, and not an independent memory in the naive sense. Matter does not necessarily survive in its ordinary form; its energetic, gravitational, and structural history, however, remains inscribed in the final state. To the question “what remains of matter?”, the model answers not by the survival of material bodies, but by the persistence of their history in the cohesive regime. Its relative share is fixed by the ratio $\mathcal{E}_M/\mathcal{E}_\mathcal{E} \approx 1/23$ (§2.2).

Residual transition clause (formerly PA-01; non-universal). If E were a state of perfect cohesion, absolutely frozen, no new state could appear and the cosmological cycle would stop. The final state would therefore preserve not only the trace of what has been, but also the architectural possibility of a new beginning. This “memory of the Exception” naturally identifies with the minimal exception ε already introduced (§2.1, eqs. 2.1–2.2): the irreducible residual asymmetry is not a defect of the final state but a necessary condition for the reopening of the cycle,

$$E \rightarrow \text{Exception} \rightarrow \text{Ignition} \rightarrow \text{Universe} \rightarrow E \quad (3.6)$$

This narrative representation of the energetic cycle does not replace the canonical critical transition $E \xrightarrow[\text{Cosmic Core}]{\text{Ignition}} m$ (§2.1).

The energetic cycle is a succession of states within open universal time. It is not a loop of time, a return to the same instant, or a perfectly identical repetition. Time remains an open universal coordinate (PA-02). LF-05 permits each new passage to retain a memory, an information content, an asymmetry, and a difference with respect to the previous state; the former PA-01 is retained only as a residual transition clause, and no longer as a first or universal law. The Ignition is an event situated *within* open time; it does not create time.

Paillette. If the exception ε disappeared entirely within E , then $E \rightarrow$ nothing: the cycle would stop. The memory of the Exception would be precisely what prevents the state of maximal cohesion from being a definitive endpoint. A speculative statement, kept as an architectural track: *the Exception is preserved across cycles.*

3.7 Status of R_{NC} : local saturation versus global scale

The two open problems named by the Tome I stabilization (v3.3) are registered here, strictly separated from the established results of this chapter.

A local saturation condition can determine a critical density, field amplitude, gradient or invariant. It does not by itself determine the global radius of the Cosmic Core. A unique value of R_{NC} additionally requires a global conserved quantity, boundary condition, matching condition, integral charge or independently derived total energy.

The general relation

$$R_{NC} = \left(\frac{3\mathcal{E}_{NC}}{4\pi\bar{\rho}_{NC}} \right)^{1/3} \quad (3.7)$$

is retained **only as a conditional identity**, valid for a spherical geometry and a defined mean density $\bar{\rho}_{NC}$. Even if a saturation mechanism fixed $\bar{\rho}_{NC}$, the total content $\mathcal{E}_{NC}$ would still have to be determined independently.

The baryonic mass is not the obligatory foundation of R_{NC} : it is the counting used by the progressive construction of the size-estimates sections above. A derivation independent of baryon counting remains possible only if another global quantity is derived first — for example: the total energy of the cycle; a cohesion charge; an integral flux; a global invariant; a topological condition; or an integration constant fixed by boundary conditions. **None of these possibilities is selected here as an active mechanism.**

The Electronic Anchoring Branch (PF-e-R1 Freeze)

The observable-domain budget constructed in §2.2 is retained as the legacy specification A,

$$\mathcal{E}_{\text{obs}}^{(A)} = 2.160 \times 10^{71} \text{ J}. \quad (3.7a)$$

Together with the anchoring density below, it defines an observable-domain mean-density scale. It does not by itself solve the global Cosmic-Core field or its boundary-value problem.

The candidate Anchoring Relation sets the critical macroscopic cohesion density through the geometric mean of the two canonical boundary regimes of State $\mathcal{A}$:

$$u_{E,\text{max}}^{\text{anc}} = \sqrt{u_{\star}^{(\text{R1})} u_{\Lambda}}, \quad (3.7b)$$

where $u_{\Lambda} = 5.2532 \times 10^{-10} \text{ J m}^{-3}$ denotes the cosmological driving background density, and $u_{\star}^{(\text{R1})}$ designates the **density of the local cohesive response of State $\mathcal{A}$ to the lightest stable charged fermion (the electron)**, rather than the internal density of the electron itself. This ontological clarification dissolves the compositeness objection ($< 10^{-18} \text{ m}$) and makes the scale involution

$$D(u) = \frac{u_{\star}^{(\text{R1})} u_{\Lambda}}{u} \quad (3.7c)$$

strictly **internal to State $\mathcal{A}$** , mapping its local saturation response directly to its cosmological infrared background.

Variational elimination of the geometric factor η . In the spatial problem (Note XVI-6), the localized matter mode $\psi(\mathbf{x})$ is confined by its own sector (electromagnetic and strong interactions) while State $\mathcal{A}$ carries a local relational response $\delta Q_{ij}(\mathbf{x}) = C_{ij}[\psi^{\dagger}\psi](\mathbf{x})$ without requiring any fundamental exclusion radius (Issue d). Under the net MIT boundary condition,

the interior ground-state profile is given by

$$\rho(r) = \left[j_0^2 \left(\frac{x_1 r}{R_e} \right) + j_1^2 \left(\frac{x_1 r}{R_e} \right) \right] \Theta(R_e - r), \quad x_1 \approx 2.042787,$$

with $R_e = \xi \hbar / (m_e c_{\mathcal{E}})$ and $\xi = \frac{4}{3} x_1 \approx 2.723716$. For the retained quadratic response, the effective participation volume is

$$V_{\text{eff}}^{(\text{R1})} \equiv \frac{(\int \rho^2 d^3x)^2}{\int \rho^4 d^3x} = f_{\rho^2} \frac{4\pi}{3} R_e^3, \quad f_{\rho^2} = 0.7734146773. \quad (3.7d)$$

The geometric ambiguity η is thereby eliminated within this branch.

Constitutive freeze of the response law (PF-e-R1). The local response verifies $\delta Q_{ij} \propto \rho$, producing the quadratic relational energy density $\varepsilon_{\mathcal{E}} \propto \delta Q_{ij} \delta Q^{ij} \propto \rho^2$ ($p = 2$). With $\lambda = 1$ in $\int \varepsilon_{\mathcal{E}} d^3x = \lambda m_e c_{\mathcal{E}}^2$, the consistent quadratic chain is

$$V_{\text{eff}}^{(\text{R1})} = 3.76952 \times 10^{-36} \text{ m}^3, \quad u_{\star}^{(\text{R1})} = \frac{m_e c_{\mathcal{E}}^2}{V_{\text{eff}}^{(\text{R1})}} = 2.171924 \times 10^{22} \text{ J m}^{-3}. \quad (3.7e)$$

Consequently,

$$u_{E,\text{max}}^{\text{anc}} = \sqrt{u_{\star}^{(\text{R1})}} u_{\Lambda} = 3.3778 \times 10^6 \text{ J m}^{-3}, \quad R_{NC}^{\text{anc}}(A) = 262\,211.6 \text{ ly}. \quad (3.7f)$$

The second number is the spherical mean-density scale obtained from $u_{E,\text{max}}^{\text{anc}}$ and $\mathcal{E}_{\text{obs}}^{(A)}$; it is not yet the radius of a solved global equilibrium.

Archived linear branch PF-e-R0. The former triplet

$$V_{\text{eff}} = 4.55696 \times 10^{-36} \text{ m}^3, \quad u_{\star} = 1.796615 \times 10^{22} \text{ J m}^{-3}, \quad u_{E,\text{max}} = 3.0721 \times 10^6 \text{ J m}^{-3}$$

belongs to the linear-response branch PF-e-R0 ($p = 1$), whose corresponding specification-A anchored scale is 270 634.7 ly. These values are preserved for the branch history but are not used in PF-e-R1.

Cosmological specifications of $\mathcal{E}_{\text{obs}}$. The legacy budget A was assembled using $u_{\Lambda}^{\text{budget}} \simeq 6.0 \times 10^{-10} \text{ J/m}^3$ and a baryonic mass scale of order 10^{53} kg , whereas the anchor uses $u_{\Lambda} = 5.2532 \times 10^{-10} \text{ J/m}^3$. The 14.2% difference is not hidden inside an error bar; it defines separate branches:

Branch	Specification	Energy $\mathcal{E}_{\text{obs}}$	R_{NC}^{anc} (ly)	Status
PF-e-R1	A: legacy vacuum; baryons	$2.160 \times 10^{71} \text{ J}$	262 211.6	Retained & frozen
PF-e-R0	A, linear response ($p = 1$)	$2.160 \times 10^{71} \text{ J}$	270 634.7	Archived
PF-e-R4	B: consistent u_{Λ} ; baryons	$1.904 \times 10^{71} \text{ J}$	251 394.9	Open
PF-e-R5	C: consistent u_{Λ} ; total matter	$2.648 \times 10^{71} \text{ J}$	280 623.9	Open

PF-e-R1 remains frozen by continuity of the register, not because specification A has been physically selected. A future selection of B or C opens a new branch and does not rewrite PF-e-R1.

Temporal cadence independence. The chain contains no occurrence of τ_e . Conditionally upon the normalization of $c_{\mathcal{E}}$, the spatial branch PF-e-R is independent of the clock selection $(\tau_Z, \tau_C, \bar{\tau}_C)$, dissolving the former degeneracy $R_{NC} \propto \sqrt{\eta\tau_e}$.

STATUS OF THE ANCHORING RESULT

PF-e-R1 freezes a conditional electronic anchoring chain and the specification-A mean-density scale $R_{NC}^{\text{anc}}(A) = 262\,211.6$ ly. The physical content of the cosmic budget and the regular global solution of the final cohesive domain remain separate calculations.

Global compactness audit: local Schwarzschild versus Cosmic Core

The specification-A energy has the mass equivalent

$$M_{\text{eq}}^{(A)} = \frac{\mathcal{E}_{\text{obs}}^{(A)}}{c^2} = 2.4033 \times 10^{54} \text{ kg}. \quad (3.7g)$$

If it were treated as an isolated mass surrounded by an asymptotically transmissive exterior, the Schwarzschild formula would give

$$R_S = \frac{2GM_{\text{eq}}^{(A)}}{c^2} = \frac{2G\mathcal{E}_{\text{obs}}^{(A)}}{c^4} = 3.5695 \times 10^{27} \text{ m} = 3.7730 \times 10^{11} \text{ ly}, \quad (3.7h)$$

and therefore

$$\frac{R_S}{R_{NC}^{\text{anc}}(A)} = 1.4389 \times 10^6. \quad (3.7i)$$

Applied without a domain restriction, the local compact-object formula would place the anchored scale more than six orders of magnitude inside its nominal Schwarzschild radius. This result is recorded explicitly.

It is not, by itself, a solution or a refutation of the Cosmic Core. The Schwarzschild profile assumes a localized source, an exterior vacuum or transmissive region, and a normalization imposed at large radius. The Cosmic Core is instead posited as the final cohesive state of the complete manifested domain; it has no external State $\mathcal{A}$ region in which $C(r) \rightarrow 0$ as $r \rightarrow \infty$ can be imposed. The local solution $C(r) = R_S/r$ is therefore not licensed for the total cosmic budget.

GLOBAL MATCHING FRONTIER

To promote $R_{NC}^{\text{anc}}(A)$ from a conditional mean-density scale to the physical radius of the Cosmic Core, RM must select the physical content of $\mathcal{E}_{\text{obs}}$, derive a regular global solution for Q_{ij} or its relevant projection, and specify the centre condition, the boundary of the manifested domain, the saturated-phase equation of state, and the matching rule appropriate to Silentium.

The tensor sector and the emergence of $w_{\mathcal{E}}$

SCOPE RESTRICTION AFTER CNL-BI1 AND v4.7.0

Equations (3.8)–(3.17) below are retained only as an auxiliary historical homogeneous cosmological reduction inherited from the earlier corpus. Their raw variable $\Gamma = \text{Tr } Q$ is not an admissible local gauge invariant, and the later bulk audit supersedes this branch as the active zero-mode closure through GHC–1, GEC–1 and HBR–1. These equations must not be inserted into CLG-1, used as the Black-Core nonlinear completion, or used to calibrate the current thermal branch.

AUXILIARY HYPOTHESIS — NOT VALIDATED. The candidate tensor sector of the mathematical corpus (V9.5.1) proposes a saturating function of Born–Infeld / k-essence type:

$$P(X) = \mathcal{A}_{\max} \tanh\left(\frac{X}{\alpha \mathcal{A}_{\max}}\right) \quad (3.8)$$

The reference three-sector structure is now written

$$\mathcal{L} = P_{\text{sat}}(X) - U_{\text{sat}}(\Gamma) + \mathcal{L}_{\Sigma}, \quad \Gamma = \text{Tr } Q. \quad (3.9)$$

This decomposition prevents the kinetic saturation, the volumetric potential, and the boundary response from being silently identified with one another.

Three conditional readings have been isolated:

- R1.** If $U_{\text{sat}} = U_{\text{sat}}(\text{Tr } Q)$, the five trace-free directions are protected from a quadratic potential mass. If U_{sat} depends on invariants such as $\text{Tr}(Q^2)$, that protection is not automatic.
- R2.** If X is the unique Lorentzian kinetic invariant, the temporal and spatial normalizations of trace-free fluctuations carry the same factor and their local propagation speed is exactly c . If the temporal and spatial sectors are split, the candidate tanh branch instead gives the conditional relation

$$\frac{c_s}{c} = \text{sech}\left(\frac{\bar{\chi}_s}{\alpha \mathcal{A}_{\max}}\right), \quad (3.10)$$

after the required normalization identity is imposed.

- R3.** Reflection symmetry of U_{sat} about the crossing centres the profile and suppresses the leading dichroic term; without that symmetry, dichroism becomes a candidate first-order signature.

The static-wall first integral also yields a quantitative existence bound. For

$$u = \frac{\chi}{\alpha \mathcal{A}_{\max}}, \quad g(u) = \mathcal{A}_{\max} [2u \text{ sech}^2 u - \tanh u],$$

the maximum occurs at

$$4u_* \tanh u_* = 1, \quad u_* = 0.5218, \quad g_{\max} = 0.3250 \mathcal{A}_{\max}. \quad (3.11)$$

Thus a monotone static wall requires

$$U(\varphi) - U(0) \leq 0.3250 A_s \mathcal{A}_{\max} \quad (3.12)$$

along the profile. Two positive algebraic branches exist below this ceiling, merge at the ceiling, and disappear above it; the branch continuous from $u = 0$ is the presently retained physical branch, while the second branch remains unexplored.

For a homogeneous one-invariant reduction of (3.9), the effective cosmological quantities are:

$$\rho_{\mathcal{E},\text{eff}} = 2X P_{\text{sat},X} - P_{\text{sat}} + U_{\text{sat}}, \quad p_{\mathcal{E},\text{eff}} = P_{\text{sat}} - U_{\text{sat}}. \quad (3.13)$$

Introducing the dimensionless excitation $u = X/(\alpha \mathcal{A}_{\max})$, the saturation function becomes $P_{\text{sat}} = \mathcal{A}_{\max} \tanh(u)$ and its derivative $P_{\text{sat},X} = \frac{1}{\alpha} \text{sech}^2(u)$. Substituting these into the effective density reveals the static-wall existence function $g(u)$:

$$\rho_{\mathcal{E},\text{eff}} = \mathcal{A}_{\max} \left[2u \text{sech}^2(u) - \tanh(u) \right] + U_{\text{sat}} = g(u) + U_{\text{sat}}. \quad (3.14)$$

The dynamic equation of state is therefore formally derived from the Lagrangian:

$$\boxed{w_{\mathcal{E},\text{eff}}(u) = \frac{\mathcal{A}_{\max} \tanh(u) - U_{\text{sat}}}{g(u) + U_{\text{sat}}}} \quad (3.15)$$

In the current cosmological epoch, the continuum excitation is small ($u \ll 1$). The first-order expansion ($\tanh(u) \approx u$, $\text{sech}^2(u) \approx 1$) gives $g(u) \approx \mathcal{A}_{\max} u$. Defining the constitutive excitation energy $E_k = \mathcal{A}_{\max} u$ and the background potential energy $E_p = U_{\text{sat}}$, the state equation reduces to:

$$w_{\mathcal{E},\text{eff}} \approx \frac{E_k - E_p}{E_k + E_p}. \quad (3.16)$$

A pure structural background ($E_k = 0$) recovers $w = -1$. The phenomenological checkpoint $w_{\mathcal{E},\text{eff}} \simeq -0.684$ is thus no longer an arbitrary fit, but the strict physical ratio of the accumulated constitutive charge to the background tension:

$$\frac{E_k - E_p}{E_k + E_p} = -0.684 \quad \implies \quad E_k \approx 0.187 E_p. \quad (3.17)$$

Today, the constitutive excitation of State $\mathcal{E}$ represents approximately 18.7% of its structural potential energy.

Any promotion of tensor saturation to the rank of active mechanism for the determination of R_{NC} must carry exactly:

PROPOSITION NON VALIDÉE — ACCORD EXPLICITE DE PIERRE REQUIS

Chapter 4

Observable Universe, visible fraction, and total energetic cycle

The chronological age of the observable Universe today is about 13.8 billion years. But this age does not necessarily coincide with its degree of overall energetic maturity.

Within the Silentium scenario, three different quantities must be distinguished:

1. **the convergence of the observable part**, that is, the degree of transformation of the content already engaged in the dynamics accessible to observation;
2. **the visible fraction of the total final content**, that is, the share of the final energetic content already included in this observable sector;
3. **the convergence of the total cosmic cycle**, which includes the entire content destined to join the cohesion state E .

4.1 Convergence of the observable part

If the characteristic convergence time of the currently modelled sector is

$$\tau_{SCF} \approx 15\text{--}20 \text{ Ga},$$

then the degree of convergence of the observable part can be written:

$$C_{\text{obs}}(t_0) = 1 - e^{-t_0/\tau_{SCF}} \quad (4.1)$$

For $t_0 = 13.8 \text{ Ga}$, one obtains:

$$C_{\text{obs}}(t_0) \approx 0.50\text{--}0.60 \quad (4.2)$$

The residual non-convergence part of this observable portion is then:

$$Y_{\text{obs}}(t_0) = 1 - C_{\text{obs}}(t_0) \approx 0.40\text{--}0.50 \quad (4.3)$$

In other words, the currently observable portion of the Universe would already be about halfway through its energetic convergence, without having reached its final state.

4.2 Visible fraction of the final content

This estimate does not necessarily mean that the total Universe, taken as a whole, has already completed half of its cycle.

The Silentium scenario leaves open the hypothesis that part of the final content of the Cosmic Core:

- is not yet observable;
- is not yet counted in the visible energy budget;
- or has not yet joined the cohesive convergence phase described by $C_{\text{obs}}(t)$.

We then introduce the energetic visibility factor:

$$f_{\text{vis}} = \frac{\mathcal{E}_{\text{obs}}}{\mathcal{E}_{\text{final}}} \quad (4.4)$$

where:

- $\mathcal{E}_{\text{obs}}$ denotes the energetic content currently observable or already integrated into the visible budget;
- $\mathcal{E}_{\text{final}}$ denotes the total energetic content destined to converge toward the final Cosmic Core.

By construction,

$$0 < f_{\text{vis}} \leq 1.$$

When $f_{\text{vis}} < 1$, this means that the currently observable sector still represents only a fraction of the total final content of the cycle.

4.3 Convergence of the total cycle

In this writing, the convergence of the total cosmic cycle can be approximated by:

$$C_{\text{tot}}(t_0) \approx f_{\text{vis}} \cdot C_{\text{obs}}(t_0) \quad (4.5)$$

This relation is not a derived fundamental law; it constitutes a **working parametrization** allowing one to distinguish:

- the energetic maturation of the **observable sector**;
- and the energetic maturation of the **total cosmic cycle**.

The factor f_{vis} therefore does not correct the internal dynamics of C_{obs} ; it corrects only the extent of the observable sector with respect to the total final content.

If, for illustration, one takes:

$$C_{\text{obs}}(t_0) \approx 0.52 \quad \text{and} \quad f_{\text{vis}} \approx 0.50,$$

then:

$$C_{\text{tot}}(t_0) \approx 0.26.$$

In this reading, the observable Universe can therefore be already advanced in its own convergence while still representing only about a quarter of the total energetic cycle.

4.4 Conceptual formulation

The observable Universe may already be halfway through its local energetic convergence while still representing only a fraction of the total cosmic cycle.

In other words:

- C_{obs} measures the convergence of the **currently modelled observable sector**;
- f_{vis} measures the fraction of the final content already included in this sector;
- C_{tot} measures the position of the system in the **global energetic cycle**.

Cosmic youth is therefore measured not only by elapsed time, nor even by the state of the observable sector alone, but by the relation between this sector and the totality of the energetic content destined to join the cohesion state E .

4.5 Status of the hypothesis

At this stage, the factor f_{vis} does not yet have an independent derivation. It must be understood as a **structural parameter** allowing two levels of the scenario to be related:

1. the observable dynamics already engaged in the visible continuum;
2. the totality of the final content called to converge toward the Cosmic Core.

Two physical readings remain open:

Reading 1 — content beyond the horizon

Part of the final content already exists, but remains beyond the horizon or beyond observational access.

Reading 2 — content not yet converted

Part of the final content has not yet entered the cohesive convergence phase described by $C_{\text{obs}}(t)$.

In both cases, $f_{\text{vis}} < 1$ means that the currently visible sector must not be confused with the complete final system.

Chapter 5

The cosmological constant as residual curvature

5.1 The pivot equation

Pillar II proposes to interpret the cosmological constant as the diluted trace of the initial cohesive curvature of the Cosmic Core. The candidate equation is:

$$\Lambda_{SCF} \sim \frac{1}{(R_{NC} \cdot D)^2} \quad (5.1)$$

where:

- R_{NC} is the radius of the Cosmic Core;
- D is a dilution / dispersion factor to be defined physically.

This relation does not yet constitute a demonstration. It plays the role of a pivot equation: it condenses the idea that a cosmological quantity observed today could preserve the geometric and energetic trace of an initial cohesion state.

5.2 Numerical check

From the observed cosmological constant, one obtains a characteristic scale:

$$L_\Lambda = \frac{1}{\sqrt{\Lambda_{\text{obs}}}} \approx 9.5 \times 10^{25} \text{ m} \approx 10^{10} \text{ ly} \quad (5.2)$$

Taking $R_{NC} \approx 2.5 \times 10^5$ light-years, one obtains:

$$D = \frac{L_\Lambda}{R_{NC}} \approx \frac{10^{10}}{2.5 \times 10^5} \approx 4 \times 10^4 \quad (5.3)$$

which leads to:

$$\Lambda_{SCF} \approx 1.1 \times 10^{-52} \text{ m}^{-2}$$

in order-of-magnitude agreement with observation.

This numerical agreement is a **consistency check**, not an independent prediction: D is obtained here from L_Λ , so relation (5.1) recovers Λ_{obs} by construction. The circularity warning attached to Λ_{SCF} (arbitration A6, Constitution V6) is constitutionally opposable: the predictive content is null as long as D has not received a definition independent of Λ .

5.3 Meaning of this relation

In this framework, Λ would no longer be an arbitrary constant added to the vacuum, but **the residual curvature of an initial cohesion state of finite radius**. The factor D would then measure the way this cohesion has distributed itself through the continuum since the Ignition.

This reading is important for Pillar II, because it shifts the question of the cosmological constant. Instead of asking “why does the vacuum have this value?”, it asks: “what initial cohesive quantity must be diluted to recover this curvature today?”

Chapter 6

Physical definition of D

The factor D appears in the relation:

$$\Lambda_{SCF} \sim \frac{1}{(R_{NC} \cdot D)^2} \quad (6.1)$$

and plays a central role in the interpretation of the cosmological constant. But a numerical result has no theoretical value as long as its physical meaning remains undetermined.

6.1 Four hypotheses

At this stage, four readings of D remain open:

Hypothesis	Definition	Reading
A — geometric	$D_L = L_\Lambda / R_{NC}$	scale ratio between the Core and the diluted Universe
B — energetic	$D_\mathcal{E} = \mathcal{E}_{NC} / \mathcal{E}_{\mathcal{E}, \text{current}}$	measure of energetic dispersion since the Core
C — cohesive	$D_\rho = \rho_E^{\text{max}} / \rho_\Lambda$	measure of cohesion loss between the initial and current states
D — memory-based	$D_{\text{mem}} \sim \mathcal{M}_{NC} / \mathcal{M}_{\text{Universe}}$	measure of dispersion of cosmic memory since the Ignition

All four readings are **dimensionless ratios**: in each, the numerator and the denominator carry the same dimensions, as required by $\Lambda_{SCF} \sim (R_{NC} D)^{-2}$ with R_{NC} a length. They are conditional constructions, not yet identified as a single demonstrated quantity. These four hypotheses are not mutually exclusive. They may designate partial readings of a single deeper quantity. The problem is therefore not only to “choose the right definition”, but to determine whether D is a simple quantity or a composite one.

Arbitration A6 (closed, Constitution V6) fixes the status of these readings: reading A (geometric) is declared **non-predictive by construction**; none of the non-circular readings (B, C, D) is privileged at this stage; the choice of a reading is an act of research programme, not of constitution. The factor D of this Pillar corresponds to the factor $D_\mathcal{E}$ of the constitutional notation. D must be dimensionless in $\Lambda_{SCF} \sim (R_{NC} D)^{-2}$; no interpretation of D becomes active without derivation, and the calibration of Λ_{SCF} from Λ_{obs} (§5.2) must not be called a prediction.

6.2 An important tension

Hypotheses A and C lead to very different orders of magnitude:

$$D^{(A)} \approx 4 \times 10^4 \quad (6.2)$$

$$D^{(C)} \approx 6.5 \times 10^{15} \quad (6.3)$$

This tension must not be masked: if D is a simple ratio of lengths, it is $\sim 10^4$; if it is a ratio of densities, it is $\sim 10^{15}$. These two numbers cannot designate exactly the same quantity, unless D has a more composite definition.

One possibility is then that D is neither purely geometric nor purely cohesive, but a **mixed quantity**, combining spatial dilution, energetic dispersion, and cohesion loss.

6.3 Dimensional reading of the tension

The preceding tension between $D^{(A)} \approx 4 \times 10^4$ (geometric) and $D^{(C)} \approx 6.5 \times 10^{15}$ (cohesive) admits a simple reading. Both quantities are dimensionless, but they do not compare the same thing: $D^{(A)} = L_\Lambda / R_{NC}$ is a ratio of *lengths*, whereas $D^{(C)} = \rho_E^{\max} / \rho_\Lambda$ is a ratio of *densities*.

The passage from a ratio of densities to a ratio of lengths is governed by the complete relation

$$D^{(C)} = q\mathcal{E} \frac{\kappa_{\text{obs}}}{\kappa_{NC}} \left(\frac{R_{\text{obs}}}{R_{NC}} \right)^3 \quad (6.4)$$

where the cubic exponent originates explicitly from the ratio of spherical volumes $V = \frac{4\pi}{3}R^3$,

$$q\mathcal{E} = \frac{\mathcal{E}_{NC}}{\mathcal{E}_{\mathbb{E},0}} \quad (6.5)$$

is the ratio of the compared energy budgets, and κ_{obs} , κ_{NC} are shape factors accounting for non-uniform density profiles. For $D^{(A)} = L_\Lambda / R_{NC}$, the bridge between the two readings is

$$\boxed{\frac{D^{(C)}}{(D^{(A)})^3} = q\mathcal{E} \frac{\kappa_{\text{obs}}}{\kappa_{NC}} \left(\frac{R_{\text{obs}}}{L_\Lambda} \right)^3} \quad (6.6)$$

The simplified form used below is allowed only under the declared hypotheses

$$q\mathcal{E} \simeq 1, \quad \kappa_{\text{obs}} \simeq \kappa_{NC},$$

which are working approximations, **not fundamental identities** (numerically, with the budgets of §2.2 in joules, $q\mathcal{E} \approx 2.16 \times 10^{71} / 2.07 \times 10^{71} \approx 1.04$). Under these hypotheses, with $R_{\text{obs}} =$

46×10^9 ly and $R_{NC} = 2.5 \times 10^5$ ly:

$$\left(\frac{R_{\text{obs}}}{R_{NC}}\right)^3 = (1.84 \times 10^5)^3 \approx 6.2 \times 10^{15} \approx D^{(C)} \quad (6.7)$$

The apparent “exponent” relating $D^{(C)}$ to $D^{(A)}$ is then

$$p = \frac{\ln D^{(C)}}{\ln D^{(A)}} \approx 3.44 \quad (6.8)$$

and the departure from the integer exponent 3 is carried by the choice of outer radius (R_{obs} against L_Λ).

6.4 Status of the factor of order 100

Under the same declared hypotheses, the residual factor evaluates arithmetically, with $R_{\text{obs}} \approx 4.6 \times 10^{10}$ ly and $L_\Lambda \approx 10^{10}$ ly, as

$$\left(\frac{R_{\text{obs}}}{L_\Lambda}\right)^3 \simeq 4.6^3 \simeq 97 \simeq 100 \quad (6.9)$$

This calculation is arithmetically correct under the adopted radii. It does not yet constitute an internal dynamical derivation.

The factor of order 100 is a conditional numerical consequence of the adopted radius definitions, the cubic volume relation, the approximate equality of the compared energy budgets and the assumed shape factors. Its dynamical origin remains to be derived from the internal expansion history of State $\mathcal{A}$.

Classification: CONDITIONAL NUMERICAL RELATION — DYNAMICAL DERIVATION OPEN.

Two conditional branches are henceforth distinguished (Arbitration Q6, §6.7): the **inverse- Λ branch** F_Λ , built on the scale $L_\Lambda \equiv \Lambda_{\text{SCF}}^{-1/2}$, and the **dynamical branch** F_∞ , built on the asymptotic expansion scale $R_\infty \equiv c/H_\infty$; both are defined in §6.5.

Neither 19 nor 100 constitutes a required value of Foundation/TRD. They are the respective consequences of two conditional scale constructions.

The different length constructions describe physical states and expansion cadences within universal cosmological time. They do not define different times and do not alter the universal temporal coordinate.

6.5 Internal derivation of the horizon ratio

The dynamical derivation that remains open is registered as follows. The observable radius is the particle horizon integrated from the Ignition — not from an automatic temporal origin $t = 0$: PA-02 imposes an open universal time, and the Ignition is an event situated within that time. Hence:

$$R_{\text{obs}}(t_0) = a(t_0) c \int_{t_{\text{Ign}}}^{t_0} \frac{dt}{a(t)} \quad (6.10)$$

With the convention $a(t_0) = 1$:

$$R_{\text{obs}} = c \int_{a_{\text{Ign}}}^1 \frac{da}{a^2 H(a)} \quad (6.11)$$

If an asymptotic radius is defined by

$$R_{\infty} = \frac{c}{H_{\infty}} \quad (6.12)$$

this definition is a **physical convention to be justified**, not an automatic consequence. The candidate dynamical ratio is then

$$\frac{R_{\text{obs}}}{R_{\infty}} = H_{\infty} \int_{a_{\text{Ign}}}^1 \frac{da}{a^2 H(a)} \quad (6.13)$$

and the complete candidate factor of the **dynamical branch** becomes

$$F_{\infty} = q\varepsilon \frac{\kappa_{\text{obs}}}{\kappa_{NC}} \left[H_{\infty} \int_{a_{\text{Ign}}}^1 \frac{da}{a^2 H(a)} \right]^3 = q\varepsilon \frac{\kappa_{\text{obs}}}{\kappa_{NC}} \left(\frac{R_{\text{obs}}}{R_{\infty}} \right)^3 \quad (6.14)$$

The **inverse- Λ branch** is defined in parallel on the scale L_{Λ} :

$$F_{\Lambda} = q\varepsilon \frac{\kappa_{\text{obs}}}{\kappa_{NC}} \left(\frac{R_{\text{obs}}}{L_{\Lambda}} \right)^3 \quad (6.15)$$

Under the conditional relation $H_{\infty}^2 = \Lambda_{\text{SCF}} c^2 / 3$ (so that $R_{\infty} = \sqrt{3} L_{\Lambda}$), the two branches obey exactly:

$$\boxed{F_{\Lambda} = 3\sqrt{3} F_{\infty}} \quad 3\sqrt{3} \simeq 5.196 \quad (6.16)$$

With the conditional values of the computation of 21 July 2026 (interface-imported $H(a)$ of the Treatise, P-02): $R_{\text{obs}}/L_{\Lambda} \simeq 4.62$, hence $F_{\Lambda} \simeq 99\text{--}103$ — status: **CONDITIONAL INVERSE-LAMBDA BRANCH — NOT A CANONICAL SCALE**. The value of order 100 follows conditionally from the inverse- Λ scale, the cubic volume relation, the energy-budget factor and the adopted geometric factors; it is not an independent prediction while L_{Λ} is constructed from Λ_{SCF} itself. And $R_{\text{obs}}/R_{\infty} \simeq 2.67$, hence $F_{\infty} \simeq 19\text{--}20$ — status: **CONDITIONAL DYNAMICAL BRANCH — PHYSICAL SCALE SELECTION OPEN**. The dynamical branch follows from the asymptotic expansion scale $R_{\infty} = c/H_{\infty}$ and from the internal integration of $H(a)$; its numerical value does not invalidate the inverse- Λ branch and does not, by itself, establish that R_{∞} is the scale entering $D_{\mathcal{A}}$. The difference between values near 19 and values near 100 does not represent a new physical amplification: it follows exactly from the cubic dependence on two length conventions differing by the factor $\sqrt{3}$.

F_∞ and F_Λ must not be announced as definitive as long as the integral has not been computed with an $H(a)$ derived from the constitutional chain (PA-09), restored here explicitly:

$$\text{EM} \rightarrow \mathcal{A} \rightarrow \text{cohesive work} \rightarrow \rho_{\mathcal{A}} \rightarrow P_{\mathcal{A}} \rightarrow H(z).$$

The ratio R_{obs}/L_Λ must never replace this chain: any derivation of the factor of order 100 must start from an $H(a)$ produced by this internal dynamics, and not from a radius chosen to reproduce an observation.

6.6 Dimensional contest: R_{NC} and the factor of order 100

1. Which global quantity fixes the scale of R_{NC} ?
2. Does tensor saturation fix a density, an invariant, or a radius?
3. Which field equations determine $X(r)$?
4. Which matching conditions define the boundary of the Core?
5. Is the radius unique, or does it belong to a family of solutions?
6. What is the internal definition of L_Λ or R_∞ ?
7. Which $H(a)$ derived from PA-09 must enter the horizon integral?
8. Is the dynamical ratio obtained actually close to 4.6?
9. Does the cube of that ratio actually give a factor close to 100?
10. What is the exact role of $q_{\mathcal{E}}$ and of the shape factors?

6.7 Decision register

Methodological rule

CONTROLLED PLURALITY OF ADMISSIBLE PATHS.

When an internal mechanism admits several mathematical representations compatible with the higher levels of Foundation/TRD, no representation is selected merely because of the numerical result it produces. The admissible paths are retained separately, with their assumptions, domains and consequences, until an independent physical mechanism, matching condition or observable prediction discriminates between them. Every retained path must preserve universal cosmological time and distinguish it from the variable local cadence of physical evolution.

This rule is methodological. It constitutes neither a new Absolute Principle, nor a new Fundamental Law, nor a new transition, nor a validation of all possible mathematical constructions. Only paths coherent with the higher levels, the units, the equations and the physical conditions are admissible.

Arbitration Q6

ARBITRATION Q6 — CONTROLLED PLURALITY OF ADMISSIBLE SCALES.

No canonical identification of $R_{NC}D_{\mathcal{E}}$ with either $L_{\Lambda} = \Lambda_{\text{SCF}}^{-1/2}$ or $R_{\infty} = c/H_{\infty}$ is adopted at the present stage. The two constructions are retained as distinct conditional branches. The inverse- Λ branch yields $F_{\Lambda} \simeq 99\text{--}103$. The dynamical asymptotic branch yields $F_{\infty} \simeq 19\text{--}20$. Under $H_{\infty}^2 = \Lambda_{\text{SCF}}c^2/3$, they obey $F_{\Lambda} = 3\sqrt{3}F_{\infty}$. Neither numerical result is used as a target for selecting the physical scale. The definitive scale must be independently derived from the state dynamics of State $\mathcal{E}$, its effective pressure, the resulting expansion cadence $H(a)$, and an explicit physical matching condition. Throughout both branches, cosmological time remains universal; only the local cadence of physical evolution varies.

Arbitration Q6 is an active methodological decision of the author, taken under explicit delegation. It modifies no Absolute Principle, no Fundamental Law and no validated transition. Any future promotion of L_{Λ} or of R_{∞} as the canonical scale must carry exactly:

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6.8 Adopted candidate definition

The most fruitful working definition at this stage is the following:

D represents the degree to which the initial cohesive energy of the Cosmic Core has distributed itself through the cosmological continuum since the Ignition.

This formulation makes it possible to keep the geometric reading, while leaving open a deeper reading in terms of cohesion or memory.

It has an important advantage: it does not require deciding immediately whether D is a ratio of lengths, of densities, of energies, of memories, or a more subtle dimensionless effective quantity. It first defines its **physical role**; in every reading, D itself remains dimensionless.

6.9 Memory-based reading of D

The most ambitious version of the model consists in interpreting D no longer merely as a scale factor, but as a **ratio of memory dilution**.

In this reading, Λ would measure not only a residual curvature; it would also measure **how far the Universe has moved away from its initial cohesion state**. In other words, the factor D would translate not only an expansion or energetic dilution, but also the **progressive dispersion of the cosmic history initially contained in the Core**.

This reading remains speculative. It is not necessary for the calculation of Λ , but it could become important if Pillar II were, later, to explicitly integrate the notion of **cosmic memory**. It constitutes a natural junction point with future developments on the memory of matter, cohesion memory, and the memory of the Exception.

Chapter 7

Primordial consistency and CMB/BAO protection

v4.7.0 STATUS

This chapter is retained as a background-level viability record of the earlier effective cosmological branch. Its u_{drive} checkpoint is not the active homogeneous closure. The current zero-mode authority is HBR–1 plus its conditional thermal/curvature extension; a new perturbative calculation must be built on that branch before CMB/BAO protection can be promoted.

7.1 Separation of the invariant background and the late dynamic reservoir

The phenomenological variable $X_{\mathcal{E}}(t)$ maps to the dynamic reservoir $u_{\text{drive}}(t)$. The invariant background $u_0 \equiv u_{\Lambda}$ and the late radiatively driven reservoir must not be conflated. In the reference normalization,

$$u_{\text{drive}}(t_0) = \frac{3c^2}{8\pi G} \alpha_X X_{\mathcal{E}}(t_0). \quad (7.0a)$$

This relation assigns physical units to the reconstructed reservoir; it does not by itself derive the primordial initial conditions.

7.2 Conditional primordial-protection criterion

At recombination, $z_{\text{rec}} \approx 1100$, the adopted stellar source is absent:

$$S_{\star}(z_{\text{rec}}) = 0. \quad (7.0b)$$

This fact alone does not force the dynamic reservoir to vanish, because a homogeneous reservoir could have been inherited from an earlier epoch. The late stellar-source branch therefore requires the explicit background conditions

$$u_{\text{drive}}(z_{\text{rec}}) = 0, \quad Q_{\text{conv}}(z \geq z_{\text{rec}}) = 0. \quad (7.0c)$$

The homogeneous expansion equation at recombination must retain radiation:

$$H^2(z_{\text{rec}}) = \frac{8\pi G}{3} [\rho_m(z_{\text{rec}}) + \rho_r(z_{\text{rec}})] + \frac{8\pi G}{3c^2} [u_0 + u_{\text{drive}}(z_{\text{rec}})]. \quad (7.0d)$$

Under Eq. (7.0c), it reduces to

$$H^2(z_{\text{rec}}) = \frac{8\pi G}{3} [\rho_m(z_{\text{rec}}) + \rho_r(z_{\text{rec}})] + \frac{8\pi G}{3c^2} u_0. \quad (7.0e)$$

An expression containing only ρ_m cannot establish protection of observables generated by the photon–baryon plasma.

Background agreement is necessary but not sufficient. Preservation of the acoustic transfer functions also requires the State $\mathcal{A}$ dynamic sector to remain unexcited at first order throughout the relevant primordial interval. A minimal sufficient set of conditions for the retained late-source branch is

$$\delta u_{\text{drive}} = 0, \quad \delta Q_{\text{conv}} = 0, \quad \Pi_{\text{drive}} = 0 \quad (z \geq z_{\text{rec}}), \quad (7.0f)$$

where Π_{drive} is the anisotropic stress carried by the dynamic reservoir.

7.3 Primordial Protection (Background Limit)

The ratio of accumulated constitutive energy from radiative work to the local matter density vanishes asymptotically in the early Universe due to the $\rho_m \propto (1+z)^3$ domination. At the recombination epoch, the absence of stellar sources forces the dynamic reservoir excitation relative to matter to analytically approach zero:

$$\lim_{z \rightarrow 1100} \frac{u_{\text{drive}}(z)}{\rho_m(z)c^2} \approx 0 \quad (7.0g)$$

PRIMORDIAL CONSISTENCY CRITERION SATISFIED

The State $\mathcal{A}$ macroscopic constitutive excitation is an emergent late-time cosmological phenomenon. The background equation natively reduces to the standard Λ CDM limit at $z \approx 1100$, formally satisfying Eq. (7.0c). The derivation of the perturbation conditions Eq. (7.0f) from the constitutive action remains the final requirement for total Boltzmann propagation.

Chapter 8

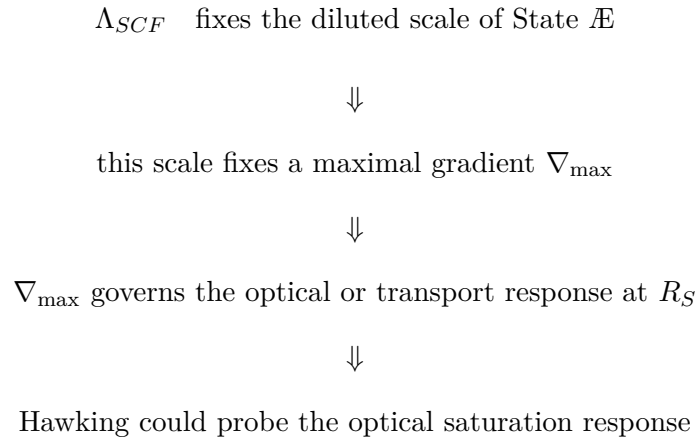
The link with Hawking

Pillar II does not merely associate Λ with the Cosmic Core. It also explores the idea that a single architecture could link:

- the maximal cohesion density $\rho_E^{\max}$;
- the maximal gradient of the continuum;
- the optical saturation surface R_S ;
- the distinct mechanical transition toward E at r_c ;
- and Hawking radiation around compact objects.

8.1 The logical chain

The proposed reasoning is the following:



This chain defines a **coherence programme**: to relate a global cosmological quantity to a local observable around Black Cores. It is distinct from the mechanical $\mathcal{A}E \rightarrow E$ transition at r_c and is not yet derived from a transport action.

8.2 The maximal gradient of State $\mathcal{A}E$

Between the Cosmic Core ($\rho_E^{\max}$) and the current Universe (ρ_Λ), the maximal gradient of the continuum can be estimated by:

$$\nabla_{\max} \sim \frac{\rho_E^{\max} - \rho_\Lambda}{L_\Lambda} \approx \frac{3.9 \times 10^6}{9.5 \times 10^{25}} \approx 4.1 \times 10^{-20} \text{ J/m}^4 \quad (7.1)$$

This quantity represents the maximal energetic slope between the cohesion state of the Core and the current diluted state of the continuum.

8.3 The Macroscopic Gradient of Cohesion

For a localized source with a transmissive exterior profile $C(r) = R_S/r$, the optical compactness gradient at the transport threshold is

$$|\nabla C(R_S)| = \frac{1}{R_S} = \frac{c^2}{2GM} \quad (7.2)$$

8.4 Hawking Temperature at the Optical Threshold

Stephen Hawking's historical formulation for the temperature of a Black Core of mass M is:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B} = \frac{\hbar c}{4\pi k_B} \left(\frac{c^2}{2GM} \right) \quad (7.3)$$

Substituting the optical compactness gradient into the formula yields the exact surface identity

$$\boxed{T_H = \frac{\hbar c}{4\pi k_B} |\nabla C(R_S)|} \quad (7.4)$$

OPTICAL STATUS OF THE HAWKING IDENTITY

Equation (7.4) belongs to the optical threshold R_S , not to the mechanical $\mathcal{A}E \rightarrow E$ wall at r_c . The former latent-heat interpretation at R_S is withdrawn in v3.10.0. RM retains the exact proportionality between T_H and $|\nabla C(R_S)|$, but a physical production mechanism must now be derived from optical or transport saturation. The identity alone does not select between competing microscopic explanations.

8.5 Complete chain

The complete chain of Pillar II can then be written:

$$\rho_E^{\max} \xrightarrow{\text{dilution}} \rho_\Lambda \xrightarrow{\text{exterior field}} C(r) \xrightarrow{C(R_S)=1} T_H \propto |\nabla C(R_S)|, \quad X_{\mathcal{R}}(r_c) = 1 \xrightarrow{\text{mechanical}} E \quad (7.5)$$

Chapter 9

The local cohesion field $C(r)$

Note on origin

This chapter integrates the results of the Research Note *Cohesion Gradients as a Candidate Origin for Emergent Gravity*, relating them explicitly to the numerical results of Pillar II (P. Benaros, *Zenodo*, June 2026, DOI: 10.5281/zenodo.20773124). In conformity with PA-08 and LF-01 (Constitution V6, decision A1), “emergent” is to be read throughout as bearing on the *transmitted manifestations* of gravity, never on the existence of the gravitational law itself.

The passage from $\rho_E^{\max}$ to the local field $C(r)$ is not a change of subject, but a change of scale. Pillar II first establishes the global structure of the scenario: Cosmic Core, maximal density, dilution, link to Hawking. The field $C(r)$ then intervenes as the local version of this architecture, that is, as the variable likely to relate the global cosmological cohesion to the gravity observed around compact objects.

In this sense, the Research Note on $C(r)$ does not constitute a separate branch: it is the local formalization of the dynamics already sketched by Pillar II.

9.1 Motivation

We have obtained a provisional estimate of the maximal cohesion energy density (under the adopted R_{NC} hypothesis):

$$\rho_E^{\max} \approx 3.9 \times 10^6 \text{ J/m}^3$$

and proposed a qualitative link with Hawking via $\nabla_{\max}$.

One link is missing, however: **how to pass from the global density of the Core to a local gradient around a Black Core?**

The answer proposed by the Research Note is the **local cohesion field** $C(r)$.

9.2 Domain of validity of the exterior branch

The local construction used in this chapter assumes simultaneously:

- (i) a localized, approximately spherical source of mass M ;
- (ii) an exterior region in the transmissive State $\mathcal{A}$ branch;
- (iii) the boundary condition $C(r) \rightarrow 0$ in the large-radius exterior;

(iv) use of the unsaturated exterior solution up to the first surface where $C = 1$.

These assumptions define a candidate local description of an isolated Black Core. They are not satisfied by the final Cosmic Core, which is posited as the cohesive state of the entire manifested domain. No total cosmic budget may therefore be inserted into the local exterior profile without first solving the separate global matching problem.

9.3 Definition of the field $C(r)$

We introduce a dimensionless scalar field:

$$C(r) \in [0, 1] \quad (8.1)$$

with the limiting interpretations:

- $C(r) \rightarrow 0$: transmissive regime, dominated by State $\mathcal{A}$;
- $C(r) \rightarrow 1$: optical and transport saturation of the exterior branch at R_S ; this is not the mechanical E -phase condition;
- $X_{\mathcal{R}}(r) \rightarrow 1^-$: mechanical saturation at r_c , where the E -rich phase begins.

A natural link with the maximal cohesion energy density is:

$$C(r) \approx \frac{\rho(r)}{\rho_E^{\max}} \in [0, 1] \quad (8.2)$$

Thus:

- the current diluted Universe satisfies $C \ll 1$;
- the Cosmic Core corresponds to the regime $C \rightarrow 1$.

Care must be taken not to confuse this **local and spatial** field $C(r)$ with the **global and temporal** convergence fractions of Chapter 4 ($C_{\text{obs}}(t)$ for the observable sector, $C_{\text{tot}}(t)$ for the total cycle): $C(r)$ is the local version, restricted to the neighbourhood of an object, whereas C_{obs} and C_{tot} describe the overall state of convergence.

9.4 From the cohesion proxy to the constrained relational field

The earlier Research Note proposed the phenomenological relation

$$g(r) = -\alpha_C \nabla C(r), \quad (8.3)$$

where $C(r)$ is a local cohesion indicator. This historical scalar precursor remains distinct from the local gravitational closure CLG-1.

The action-based channel is now carried by the gauge-invariant relational scalar

$$\mathcal{G}[q] = \nabla^2 \text{Tr } q - \partial_i \partial_j q_{ij}, \quad \mathcal{G}[q] = \sigma \rho_m, \quad (8.3a)$$

with the observable potential

$$\Psi_{\mathcal{A}} = \sigma \Lambda, \quad Z\sigma^2 = 8\pi G. \quad (8.3b)$$

Its static non-homogeneous limit is

$$\nabla^2 \Psi_{\mathcal{A}} = 4\pi G \rho_m. \quad (8.3c)$$

The field $C(r)$ may therefore serve as a cohesion proxy, while q_{ij} and Λ carry the conditional local quadratic closure. State $\mathcal{A}$ remains homogeneous in its fundamental density: the local object is relational, not a spatial deformation of u_0 .

The reading in Foundation/TRD language is the following:

- E provides cohesion;
- State $\mathcal{A}$ provides physical transmission;
- Q_{ij} records the local relation among sources;
- the constrained scalar extracted by $\mathcal{G}[q]$ supplies the Newtonian potential for $k \neq 0$.

The updated chain is

$$\text{matter source} \longrightarrow \mathcal{G}[q] = \sigma \rho_m \longrightarrow \Psi_{\mathcal{A}} = \sigma \Lambda \longrightarrow \text{Poisson manifestation.}$$

NEWTON'S LIMIT IS NOW CONSTRAINED

The old scalar equation named the intuition. CLG-1 supplies the spatial gauge invariant, the constraint and the unique normalisation. The measurable bridge is now $Z\sigma^2 = 8\pi G$. The result is conditional on the local quadratic action and is not a nonlinear strong-gravity derivation.

9.5 From Cohesion to the Metric Tensor

Because conventional gravitational potential is negative and $C(r)$ is defined as a positive absolute fraction, the phenomenological coupling must be strictly negative. Setting $\alpha_C = -c^2/2$ yields the absolute cohesion potential for the isolated object:

$$\Psi_{\mathcal{A}}(r) = -\frac{c^2}{2} C(r) \quad (8.4)$$

Injecting this intrinsic RM potential into the weak-field time component of the optical metric yields:

$$g_{00} = 1 + \frac{2\Psi_{\mathcal{E}}}{c^2} = 1 - C(r) \quad (8.5)$$

9.6 The Revised Schwarzschild Relation

Pillar II already reinterprets the Schwarzschild factor not as a time dilation, but as a local indicator of the state of the continuum. Within the isolated-source exterior branch and only within that branch, the Poisson solution and the weak-field optical correspondence give

$$\Psi_{\mathcal{E}}(r) = -\frac{GM}{r}, \quad C(r) = \frac{2GM}{rc^2} = \frac{R_S}{r}, \quad g_{00} = 1 - C(r). \quad (8.6)$$

The corresponding RM cadence factor is therefore

$$\mathcal{C}_{\text{RM}}(r) = \sqrt{1 - C(r)} = \sqrt{1 - \frac{R_S}{r}} = \sqrt{1 - \frac{2GM}{rc^2}}. \quad (8.6a)$$

REVISED SCHWARZSCHILD RELATION

RM retains the measurable Schwarzschild factor while changing its physical reading. The universal time coordinate is not slowed: $\mathcal{C}_{\text{RM}}(r)$ is interpreted as the local cadence factor of matter and matter-coupled processes on this branch. This named relation is used throughout Book II as a conceptual signature. Its strong-field completion remains subject to the explicit non-repair and falsification clauses of this work.

9.6.1 Cumulative saturation closure

The local cadence factor by itself does not encode the full constitutive history between a radius r and the transmissive exterior. To keep that distinction explicit, Book II introduces the following candidate cumulative closure:

$$N_{\text{RM}}(r) = \frac{P\sqrt{f(r)}}{Q(r)} \exp\left[-\int_r^\infty \mathcal{S}(s) ds\right]. \quad (8.6b)$$

Here $\sqrt{f(r)}$ carries the local cadence contribution, while the exponential factor records the cumulative constitutive saturation between r and the asymptotically transmissive exterior. The prefactor $P/Q(r)$ retains the normalization and local constitutive response of the branch. In the present edition, Eq. (8.6b) is a **candidate constitutive closure**: it is not promoted to a derived law until $\mathcal{S}$, its admissible domain, the boundary conditions and the strong-field matching are fixed independently.

LOCAL CADENCE AND CUMULATIVE SATURATION

The Revised Schwarzschild Relation identifies the local cadence factor. Equation (8.6b) adds a distinct non-local constitutive memory through $\exp[-\int_r^\infty \mathcal{S}(s) ds]$. The two expressions must not be conflated: one is local and observationally anchored on the isolated-source branch; the other is a candidate closure for accumulated saturation and remains falsifiable.

LOCAL HORIZON CORRESPONDENCE

For a localized spherical source surrounded by a transmissive exterior, $C(R_S) = 1$ reproduces the Schwarzschild surface as a local correspondence. This statement is not a global identity and cannot be applied to $\mathcal{E}_{\text{obs}}$ or to the complete Cosmic Core. The geometric horizon $r = R_S$ is exactly the physical surface where the macroscopic metric ceases to act because the continuum has fully saturated into the cohesive state E .

9.7 The critical gradient G_{crit}

The model assumes that State $\mathcal{A}E$ has a domain of stability. When the local energetic gradient exceeds a certain critical threshold, the continuum tips over into a cohesive regime:

$$G < G_{\text{crit}} \quad \Rightarrow \quad \text{stable transmissive regime} \quad (8.7)$$

$$G \geq G_{\text{crit}} \quad \Rightarrow \quad \text{transition toward } E \quad (8.8)$$

A singularity would then not be a fundamental physical object, but **the mathematical symptom of a regime transition still poorly described.**

Chapter 10

Cosmic Memory and Thermodynamic Evaporation

By dematerializing the geometric singularity into a transition surface ($C = 1$) for the isolated source, RM reproduces the standard thermodynamics of that surface entirely within its own continuum mechanics. What follows is a consistency result, not a resolution of the information paradox; the distinction is stated explicitly at the end of the chapter.

10.1 Thermodynamics of the Saturation Wall and Entropy

The boundary $C(r) = 1$ acts as a phase transition interface. Mass traversing this boundary undergoes a state conversion governed by the first law of thermodynamics:

$$c^2 dM = T_H dS \quad (9.1)$$

Substituting the derived Hawking temperature $T_H = \frac{\hbar c}{4\pi k_B} |\nabla C(R_S)|$, the differential structural entropy is:

$$dS = \frac{c^2 dM}{T_H} = \frac{8\pi G k_B}{\hbar c} M dM \quad (9.2)$$

Integrating over the total mass yields the capacity of structural storage in state E :

$$S = \frac{4\pi G k_B}{\hbar c} M^2 \quad (9.3)$$

Expressed in terms of the geometric surface area $A = 4\pi R_S^2$:

$$\boxed{S = \frac{k_B c^3}{4G\hbar} A} \quad (9.4)$$

WHAT THIS CHAPTER ESTABLISHES, AND WHAT IT DOES NOT

The algebra of (9.1)–(9.4) is exact and has been verified numerically: for one solar mass, $T_H = 6.170074 \times 10^{-8}$ K is obtained identically from the surface gravity and from $(\hbar c/4\pi k_B)|\nabla C(R_S)|$, and $S = 4\pi G k_B M^2/\hbar c$ equals $k_B c^3 A/4G\hbar$ to machine precision. The RM content of this chapter is the identification $C = -2\Psi_{\mathcal{A}}/c^2$, which follows from the Poisson limit derived in *Who Pays for Gravity?*. Given that identification, $|\nabla C(R_S)| = 2\kappa/c^2$, and the standard black-hole thermodynamics follows. Steps (9.1) to (9.4) themselves contain no RM-specific ingredient: they are the 1974 Bekenstein–Hawking integration, which returns $S = A/4$ in Planck units as soon as T_H is supplied. This is therefore a **consistency result**: RM recovers the standard result from its own potential, and does not contradict it. It is **not** a resolution of the information paradox. That paradox concerns unitarity — whether the quantum state of infalling matter is recoverable from the outgoing radiation — and an area-proportional entropy accompanying thermal radiation is part of the *statement* of the problem, not its answer. **Status T1**: conditional on the RM identification of $C(r)$, and reproducing a known result rather than replacing it.

Reproducibility record for PF-e-R1

The retained spatial branch is identified by

$$\text{PF-e-R1} = \text{PF-e}(\lambda = 1, p = 2, \mathcal{E}_{\text{obs}} = A). \quad (10.1)$$

Its numerical chain can be reconstructed from the following frozen inputs:

Quantity	Value	Role
f_{ρ^2}	0.7734146773	quadratic participation factor
$V_{\text{eff}}^{(\text{R1})}$	$3.76952 \times 10^{-36} \text{ m}^3$	electronic response volume
m_e	$9.1093837015 \times 10^{-31} \text{ kg}$	microscopic anchor
u_{Λ}	$5.2532 \times 10^{-10} \text{ J/m}^3$	macroscopic anchor
$\mathcal{E}_{\text{obs}}^{(A)}$	$2.160 \times 10^{71} \text{ J}$	frozen legacy budget
1 ly	$9.46073047258 \times 10^{15} \text{ m}$	output conversion

The calculation is

$$u_{\star}^{(\text{R1})} = \frac{m_e c^2}{V_{\text{eff}}^{(\text{R1})}} = 2.171924 \times 10^{22} \text{ J/m}^3, \quad (10.2)$$

$$u_{E,\text{max}}^{\text{anc}} = \sqrt{u_{\star}^{(\text{R1})} u_{\Lambda}} = 3.3778 \times 10^6 \text{ J/m}^3, \quad (10.3)$$

$$R_{NC}^{\text{anc}}(A) = \left(\frac{3\mathcal{E}_{\text{obs}}^{(A)}}{4\pi u_{E,\text{max}}^{\text{anc}}} \right)^{1/3} = 262\,211.6 \text{ ly}. \quad (10.4)$$

Changing p , λ or the specification of $\mathcal{E}_{\text{obs}}$ changes the branch; it is not treated as a statistical perturbation of PF-e-R1.

Calculation frontier of Pillar II

The equations of Pillar II expose nine quantitative frontiers. Two weak-field items are closed conditionally at the level calculated. N-Æ-05 protects the cadence origin of the minimal CNL-2D branch under one explicit constitutive principle, and N-Æ-06 conditionally closes the gravitational part D-Æ-04A on the regular local branch. D-Æ-02 and the residual microscopic-matter audit D-Æ-04M remain open. Historical frontier statements are retained rather than deleted, in accordance with the append-only discipline.

1. Cassini and the spatial optical operator — closed at first order in v3.11.0

The question as posed in v3.10.1 was: can the principal eikonal symbol of the electromagnetic operator be derived from the admissible relational variables and yield the observed coefficient γ_{PPN} without identifying q_{ij} with a spatial metric and without fitting an optical parameter? Note N-Æ-02 answers it. The symbol is derived, no optical parameter is introduced, and $\gamma_{\text{PPN}} = \psi/C = -\text{sgn}(\sigma)$ takes only the values ± 1 ; the branch $\sigma < 0$ gives $+1$ exactly. What remains open on this item is not the calculation but its foundation: the arbitration of debt D-Æ-02, and an explicit Dirac analysis with electromagnetic sources confirming that a coupling $\lambda \neq 1$ genuinely destroys the degree count.

2. β_{PPN} and the preferred-frame parameters α_1, α_2 — closed conditionally in v4.0.0

The consolidated N-Æ-02 gives $\beta_{\text{PPN}} = 1$ from the areal–isotropic radius conversion and independently from the static minimal CNL-2D branch. The universal sourced gauge coupling gives $\alpha_1 = \alpha_2 = 0$. N-Æ-05 now closes cadence minimality conditionally by deriving $A = P/(4Nc_{\text{E}}^2)$, $B = P/(12Nc_{\text{E}}^2)$, $C = PN$ and, under PCP, $D = V = 0$, without using a PPN observable. N-Æ-06 now supplies the universal matter interface and the first complete local nonlinear gravitational Dirac audit. The remaining priority is D-Æ-04M: explicit microscopic validation for matter sectors whose canonical formulation may alter the primary constraints. Failure withdraws the affected value; no second normalisation, sign or PPN coefficient may be introduced.

3. Size of the Cosmic Core

Can D_{NC} be derived from an internal dynamics of the model rather than introduced as a working hypothesis?

4. Status of ρ_E^{max}

Does the provisionally estimated maximal energy density correspond to a stable property of the state E , or does it still depend too strongly on the hypotheses about the diameter of the Core?

5. Definition of D

Is the factor D geometric, energetic, cohesive, memory-based, or a combination of these four readings?

6. Normalization and observational closure

Can $X_{\mathbb{E}}^{\text{notebook}}$ be mapped to the physical density $\rho_{\mathbb{E}}$, the source–coupling degeneracy be lifted, and the resulting model be tested with complete chronometer covariance, BAO, CMB, perturbations, and structure growth?

7. Primordial perturbative closure

Can Eqs. (7.0c) and (7.0f) be derived from the constitutive action and propagated through a CMB/BAO Boltzmann calculation?

8. Nonlinear completion of CLG-1

Does CNL-BI1 preserve the six first-class and ten second-class constraints on a non-zero background and keep a unique tensor front? Does the regulated variation of CNL-BI1 together with the finite mechanical phase wall produce a dimensionally homogeneous global junction that reduces locally to $[\Pi^r_r]_{r_c^-}^{r_c^+} = 2\tau_{\Sigma}/r_c$, derives τ_{Σ} without calibration, preserves the candidate scale $r_c \propto M^{1/3}$, and admits a stable regular spherical solution? The formerly printed expression $\mathcal{B}_{\text{BI}} + 8\pi r_c \tau_{\Sigma} = 0$ is withdrawn by audit A-DSB-1-01 and is not a target equation.

Founding statements of Pillar II

- CLG-1 yields the Poisson equation for $k \neq 0$ under the single matching $Z\sigma^2 = 8\pi G$.
- The same constrained quadratic action yields $E_{12} = -Gm_1m_2/r$ and $d(K+E_{12}) = 0$.
- In the isotropic stellar representative, $\psi = \Phi[q]$ and, on the stated weak branch, $\psi = -\text{sgn}(\sigma)C$.
- The same local closure gives $u_{\text{rel}} = -|\nabla\Psi_{\mathcal{A}}|^2/(8\pi G)$ and the inverse-Maxwell stress whose divergence is $\rho_m\mathbf{g}$.
- An ordinary main-sequence star is a non-saturated relational configuration: on the retained Planck-length branch, the solar core gives $X_{\mathcal{R}} \simeq 7.1 \times 10^{-181}$.
- RM presently derives the weak optical-time coefficient but not the spatial eikonal coefficient. Cassini therefore defines a rejection calculation, not a claimed success: the naive identification $\delta_{ij} + q_{ij}$ as an optical metric is forbidden.
- (v3.11.0, superseding the preceding statement on the selected branch, which is retained unaltered.) The spatial eikonal coefficient is derived in N-Æ-02. The electromagnetic principal symbol is $\mathfrak{P} = \omega^2/(N^2c_{\mathcal{A}}^2) - g^{ij}[q]k_ik_j$, giving $n^2 = (1+\psi)/(1-C)$ and $\gamma_{\text{PPN}} = \psi/C = -\text{sgn}(\sigma) \in \{-1, +1\}$. The v3.10.1 prohibition was conditional on $\sigma > 0$ (audit A-Æ-02); on $\sigma < 0$ the coefficient is $\gamma_{\text{PPN}} = +1$ exactly.
- (v4.0.0.) On the same selected branch, the physical-areal to isotropic-radius conversion and the minimal static completion give $\beta_{\text{PPN}} = 1$, while the universal sourced matter coupling gives $\alpha_1 = \alpha_2 = 0$. These values are conditional on debts D-Æ-02, D-Æ-03 and D-Æ-04 and are withdrawn, not fitted, if those debts fail.
- (v4.1.0.) N-Æ-05 introduces the Pure Cadence Principle and conditionally closes D-Æ-03. The universal cadence translation $d\tau_{\text{cad}} = N dT$ fixes $A = P/(4Nc_{\mathcal{A}}^2)$, $B = P/(12Nc_{\mathcal{A}}^2)$ and $C = PN$; PCP fixes $D = V = 0$. The whole Taylor tower is thereby frozen before D-Æ-04. If a later microscopic or Dirac calculation requires another cadence function, PCP is withdrawn and D-Æ-03 reopens rather than being repaired.
- (v4.2.0.) N-Æ-06 tests the frozen N-Æ-05 branch with a universal matter interface and the nonlinear local Dirac algorithm. On the regular $k \neq 0$ elliptic branch, the scalar sector contributes four second-class constraints and the minimal constrained antisymmetric completion contributes three second-class pairs; together with six spatial first-class constraints this gives $N_{\text{grav}} =$

$(26 - 2 \times 6 - 10)/2 = 2$. D-Æ-04A is closed conditionally; D-Æ-04M remains open and narrowed to explicit microscopic matter sectors that may modify the canonical structure. No new coefficient is introduced to preserve the count.

- The trace-free coefficient $\lambda = 1/3$ makes the conformal momentum primary in the minimal CNL-2D candidate. The associated elliptic block explains both the local restriction $k \neq 0$ and the homogeneous kernel $\mathcal{K} = 3H(T)$; it does not determine $H(T)$.
- RM contains no continuous optical parameter. γ_{PPN} cannot be fitted to any value other than ± 1 ; this rigidity, not the agreement with Cassini, is the falsifiable content of the optical closure.
- The sign of σ is unobservable in the pure relational sector, which is invariant under $(q, B, \Lambda, \sigma) \rightarrow (-q, -B, -\Lambda, -\sigma)$, and becomes physical exactly when the electromagnetic sector is coupled.
- The optical closure is conditional on debt D-Æ-02: the labelling symmetry cannot be both a gauge redundancy and a physical individuation of the points of $\Omega_{\text{Æ}}$.
- The local gravitational relation does not deplete the homogeneous fundamental density u_0 ; the bridge to the universal reservoir remains open.
- The Dirac matrix has $\det \Omega_{\text{SC}} = \frac{B^2 \mu_A^6}{16} (k^2)^{10}$ and leaves two local propagating tensor polarisations when $B\mu_A k \neq 0$.
- The nonlinear candidate CNL-BI1 uses only relational curvature invariants and its subtracted Born–Infeld term begins at $O(q^4)$; it therefore preserves every CLG-1 result at quadratic order.
- CNL-BI1 supplies a finite-density, infinite-response barrier at $X_{\mathcal{R}} = 1$, but a regular Black-Core solution and its phase surface remain to be derived.
- The Black Core carries two frozen candidate scales: $R_S \propto M$ for its optical transport surface and $r_c \propto M^{1/3}$ for its mechanical phase boundary.
- The law $r_c \propto M^{1/3}$ makes M/r_c^3 universal and gives the E -rich core the scaling signature of a three-dimensional incompressible volume.
- For $X_{\mathcal{R}} = (r_c/r)^6$, the Born–Infeld bulk energy and algebraic scalar-stress integrals are exactly $(\pi - 3)/6$ and $(5 - \pi)/6$; the corresponding normalized coefficients sum to one.
- Audit A-DSB-1-01 withdraws the formerly printed global equation $\mathcal{B}_{\text{BI}}(r_c) + 8\pi r_c \tau_{\Sigma} = 0$ as dimensionally inconsistent. The local pressure condition remains admissible; the regulated global wall–flux junction and its mass scaling are open.

- Within the isotropic uniform-core reduction, the total mass cancels from ρ_{res} and the residual fraction is $\varepsilon_M = 2\sqrt{\pi G Z}/c_{\text{Æ}}^2$.
- Maximal cohesion does not necessarily imply minimal volume.
- The Cosmic Core is posited as an extended state of cohesion, not a singular point; its regular global solution remains to be derived.
- The final Cosmic Core has zero global angular momentum: $J_{\text{final}} = 0$.
- Historical effective cosmological checkpoints are retained as viability tests only; they are not current fundamental density ratios or derived cosmic parameters.
- The local field $C(r)$ establishes the macroscopic observable metric $g_{00} = 1 - C(r)$ for an isolated source within the stated ansatz (T1).
- The Hawking-temperature identity is recovered at the optical threshold R_S through $T_H \propto |\nabla C(R_S)|$ (T1); its former identification with mechanical latent heat is withdrawn.
- Bekenstein–Hawking entropy is recovered from the optical surface $C = 1$ and the RM identification of $C(r)$ (T1, consistency result; the unitarity problem is not addressed).
- The State Æ dynamic reservoir asymptotically vanishes at high redshift ($u_{\text{drive}} \rightarrow 0$ at $z = 1100$), satisfying the Primordial Protection background limit (T2).
- The scale involution $D(u) = \frac{u_* u_\Delta}{u}$ is structurally derived from a Hamiltonian saturation of type Born-Infeld.
- The fundamental invariant anchor $\dot{u}_0 = 0$ isolates the background density from the dynamic reservoir expansion.
- L'électron définit l'ancrage microscopique universel du continuum selon le Principe de Moindre Saturation.
- The historical effective equation-of-state checkpoint belongs to the superseded radiative-expansion model; the active homogeneous authority is HBR–1 plus its conditional thermal extension.
- PF-e-R1 fixes the conditional specification-A mean-density scale $R_{NC}^{\text{anc}}(A) = 262\,211.6$ ly; it does not yet fix the physical global radius.

Constitutional status of the main statements

Statement	Status
Time as an open universal coordinate	Absolute Principle
Fundamental domain $T \in \mathbb{R}$, $\mathbf{x} \in \Omega_{\mathcal{E}}(T) \subset \mathbb{R}^3$	Foundational RM kinematics; no fourth physical dimension and no four-dimensional measure in the local action
Existence of State $\mathcal{E}$	Absolute Principle
Silencium as an ideal limit outside the manifested Universe	Absolute Principle / canonical definition
$\mathcal{E} \rightarrow \mathcal{E}$	Constitutive RM transition postulate; microscopic phase law, efficiency and entropy production remain open
Legacy chain $\mathcal{E} \rightarrow P_{\mathcal{E}} \rightarrow H(z)$	Historical schematic only; current architecture separates homogeneous bulk response H , boundary production g_{∂} and transfer to observables
PS-0: $E_{\gamma}^2 - \kappa_S^2 c_{\mathcal{E}}^2 p_{\gamma}^2 = 0$ and $Q^{\text{sat}} = \kappa_S H_{\mathcal{E}} u_{\gamma}$	Historical radiative ansatz; not the active thermal/redshift ledger after v4.7.0
$w_{\mathcal{E},\text{eff}} \simeq -0.684$	Historical effective-model checkpoint; superseded as the active homogeneous closure by HBR-1 and retained only for branch provenance
Existence of a fundamental density limit of E	Absolute Principle
$u_{E,\text{max}}^{\text{anc}} = 3.3778 \times 10^6 \text{ J m}^{-3}$	Conditional numerical normalization of the quadratic PF-e-R1 anchoring branch
Cosmic Core physical radius	Open global field, budget-selection and boundary-matching problem
Effective quadratic participation volume	Derived within the frozen electronic profile and quadratic response law; eliminates η in PF-e-R1
$V_{\text{eff}}^{(\text{R1})} = 3.76952 \times 10^{-36} \text{ m}^3$	Retained conditional specification-A mean-density scale; not a solved global radius
PF-e-R1: $\varepsilon_{\mathcal{E}} \propto \rho^2$, $\lambda = 1$, $R_{NC}^{\text{anc}}(A) = 262\,211.6 \text{ ly}$	Specification open: A is the frozen legacy input; B and C are distinct coherent candidate branches
Observable-domain budget $\mathcal{E}_{\text{obs}}$	Structurally derived from the state equation $u(X)$ (Born-Infeld type)
Involution $D(u) = u_{\star} u_{\Lambda}/u$	Justified via the Principle of Minimal Saturation
Electron as universal microscopic anchor	Internal reconstruction of the present driving density; dynamical
$u_{\text{drive},0}^{\text{RM}} = 5.510 \times 10^{-10} \text{ J m}^{-3}$	Strictly invariant background density decoupled from the dynamic reservoir, proving the absolute anchor u_{Λ}
$u_0 = u_{\Lambda}$ background anchor	Controlled plurality of dimensionless candidate readings
D	Candidate hypothesis
$g = -\alpha_C \nabla C$	Map between the physical scalar cohesion field and the macroscopic observable metric for an isolated source
$g_{00} = 1 - C(r)$ and $C(R_S) = 1$	obtained under the stated ansatz (T1)

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Statement	Status
$T_H = \frac{\hbar c}{4\pi k_B} \nabla C(R_S) $	Exact identity at the optical threshold (T1); the production mechanism must be derived from optical or transport saturation and is not identified with the mechanical wall at r_c
$S = \frac{k_B c^3}{4G\hbar} A$	T1 — standard black-hole entropy reproduced from the RM identification $C = -2\Psi_{\mathcal{E}}/c^2$; not a resolution of the unitarity problem
$\mathcal{G}[q] = \nabla^2 \text{Tr } q - \partial_i \partial_j q_{ij}$	Spatially gauge-invariant local scalar; automatically annihilates the homogeneous mode
$\nabla^2 \Psi_{\mathcal{E}} = 4\pi G \rho_m$	Conditional theorem of CLG-1 for $k \neq 0$ under $Z\sigma^2 = 8\pi G$
$E_{12} = -Gm_1 m_2 / r$	Derived interaction energy in the same constrained quadratic action
$\det \Omega_{\text{SC}} = \frac{B^2 \mu_A^6}{16} (k^2)^{10}$	Rank-ten second-class block for $B\mu_A k \neq 0$; with six first-class constraints, leaves two local modes
$q_{ij} = \psi \delta_{ij} \Rightarrow \psi = \Phi[q]$	Exact identity in the isotropic representative for $k \neq 0$ and the stated decaying boundary conditions
$\psi = -\text{sgn}(\sigma)C$	Conditional weak-branch identity following from $Z\sigma^2 = 8\pi G$ and $ \sigma = 16\pi G/c_{\mathcal{E}}^2$
$u_{\text{rel}} = - \nabla \Psi_{\mathcal{E}} ^2 / (8\pi G)$	Derived static relational field-energy density of the isotropic CLG-1 reduction
$\Pi_{ij} = -(4\pi G)^{-1} (g_i g_j - \delta_{ij} g^2 / 2)$	Derived relational stress; radial compression and transverse tension, with $\partial_j \Pi_{ij} = \rho_m g_i$. This corrects the former reversed verbal assignment
$X_{\mathcal{R}}^{\text{solar core}} \simeq 7.1 \times 10^{-181}$ for $\ell_{\star} = \ell_P$	Conditional scale audit: an ordinary solar-type star does not reach the retained nonlinear saturation barrier
γ_{PPN} from the spatial eikonal operator	Open priority calculation. The temporal coefficient alone supplies only the Newtonian half of the light response; q_{ij} is not to be identified with an optical spatial metric
$\mathfrak{P} = \omega^2 / (N^2 c_{\mathcal{E}}^2) - g^{ij} [q] k_i k_j$	Principal eikonal symbol of the electromagnetic operator, derived in N-Æ-02 from relabelling covariance, electromagnetic gauge invariance and the Exclusion Interaction (historical PS-0 label). Supersedes the preceding row on the selected branch; the preceding row is retained
$-\frac{1}{2} q_{ij} \Sigma^{ij}, \lambda = 1$	Unique relabelling-covariant electromagnetic coupling, Σ^{ij} the Maxwell stress tensor. Verified symbolically. Conditional on the gauge reading of the labelling symmetry (D-Æ-02)
$n^2 = (1 + \psi) / (1 - C)$	Exact optical index in the isotropic representative; extrapolation to $C \rightarrow 1$ is not licensed by the quadratic branch
$\gamma_{\text{PPN}} = \psi / C = -\text{sgn}(\sigma)$	Derived; admits only the values ± 1 . No continuous optical parameter exists in RM

Continued on the next page

Statement	Status
$\text{sgn}(\sigma) = -1$	Selector adopted in v3.11.0, giving $\gamma_{\text{PPN}} = +1$ exactly. Fixed by internal consistency with the DSB-1 optical threshold and by the qualitative sign of the solar time delay, not by the Cassini number. One discrete bit, four-fold over-determined; conditional, with the residual circularity of the DSB-1 argument recorded in N-Æ-02 §XI
$\beta_{\text{PPN}} = 1$	Conditional theorem of v4.0.0: derived from the physical-areal to isotropic-radius conversion and independently reproduced by the minimal constant-coefficient CNL-2D static branch; no second-order coefficient fitted
$\alpha_1 = \alpha_2 = 0$	Conditional theorem of the universal first-order matter coupling and three-dimensional material momentum balance; complete microscopic matter action still open
$S_{\text{CNL-2D}}^{\min}$	Minimal constant-coefficient two-derivative completion candidate; recovers CLG-1 at quadratic order. Not a unique-completion theorem
$\pi = g_{ij}\pi^{ij} \approx 0$ at $\lambda = 1/3$	Primary conformal constraint of the minimal candidate; nonlinear two-mode count remains conditional on the complete Dirac matrix
$\mathcal{K} = 3H(T)$ at $k = 0$	Homogeneous kernel permitted by the local elliptic chain. The reduced zero-mode rate is now supplied conditionally by HBR-1; boundary production remains a distinct sector
HBR-1	Formal result of the reduced $U = 0$ homogeneous branch under the declared matter-radiation ledger; no imported Friedmann equation or fitted homogeneous response coefficient
Thermal/Maria completion	Conditional homogeneous extension with $\varepsilon_{\mathbb{E}}^{\text{th}}$ and $U(X)$; UC-1 is a postulate and Maria-limit $\rightarrow EM \rightarrow \mathbb{E}$ remains a working microscopic bridge
P-CIN-DIRAC	Local canonical health result: regular momentum inversion, positive convex Hamiltonian, positive kinetic Hessian and non-kinetic lapse. Full field constraint preservation and hyperbolicity remain open
D-Æ-02	Open conceptual debt: gauge redundancy versus physical individuation of the labels of $\Omega_{\mathbb{E}}$. Conditions the optical closure and the degree count alike
D-Æ-03	Open constitutive debt: derive cadence minimality or classify and exclude all admissible higher functions of the physical field N
D-Æ-04	Open dynamical debt: compute the complete nonlinear Dirac matrix with the microscopic matter action
N-Æ-05	Superseding v4.1.0 entry: PCP fixes the full two-derivative cadence functions and closes D-Æ-03 conditionally; the old debt row is retained as historical state

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Statement	Status
D-Æ-04 (v4.1.0)	Immediate priority: test the frozen N-Æ-05 branch with the complete nonlinear Dirac matrix and microscopic matter interface
$\mathcal{R}^{(1)}[\gamma] = -\mathcal{G}[q]$	Derived linear identity linking the CLG-1 scalar to the nonlinear relational curvature
$X_{\mathcal{R}} = \ell_{\star}^4 \mathcal{R}_{ij} \mathcal{R}^{ij}$	Minimal positive gauge-invariant curvature norm of CNL-BI1; constitutive length $\ell_{\star}$ remains to be fixed independently
$\mathcal{U}_{\text{BI}} = u_{\star} [1 - \sqrt{1 - X_{\mathcal{R}}} - X_{\mathcal{R}}/2]$	Nonlinear candidate; begins at $O(q^4)$ and has divergent marginal response at $X_{\mathcal{R}} = 1$; full nonlinear Dirac and spherical closures open
$R_S = 2GM/c_{\mathbb{E}}^2$ and $r_c = 6^{1/6} (\ell_{\star}^2 GM/c_{\mathbb{E}}^2)^{1/3}$	Revised double-scale Black-Core candidate: optical transport surface proportional to M , mechanical phase boundary proportional to $M^{1/3}$
$M/r_c^3 = c_{\mathbb{E}}^2/(\sqrt{6}G\ell_{\star}^2)$	Derived incompressible-volume scaling of the E -rich core; microscopic equation of state and regular solution remain open
$\int_1^{\infty} y^2 F(y^{-6}) dy = (\pi - 3)/6$ and $\int_1^{\infty} y^2 [4XF' - 3F]_{X=y^{-6}} dy = (5 - \pi)/6$	Exact Born-Infeld bulk identities; both contributions scale as $u_{\star} r_c^3 \propto M$; boundary flux excluded
$\rho_{\text{res}} = 3\varepsilon_M c_{\mathbb{E}}^2/(4\pi\sqrt{6}G\ell_{\star}^2)$	Derived cancellation of the total mass within the regular uniform-core reduction
$\varepsilon_M = 2\sqrt{\pi GZ}/c_{\mathbb{E}}^2$	Conditional residual-memory identity; assumes core isotropy, $X_{\mathcal{R}} \rightarrow 1^-$, uniform residual density and continuation of the CLG-1 coupling
$0 < Z \leq c_{\mathbb{E}}^4/(4\pi G)$	Derived admissibility bound from $0 \leq \varepsilon_M \leq 1$ and the positive propagating normalization $Z > 0$
$ \sigma = 16\pi G/c_{\mathbb{E}}^2 \Rightarrow \varepsilon_M = \sqrt{2}/4$	Parameter-free consequence on the independently normalized weak-field branch; not a derivation of that normalization
$M_{\text{ext}} = M$ and $\rho_{\text{rel}}^{(\text{core})} = \rho_{m,\text{res}}$	OCH-1 ontological source separation: complete mass controls the exterior tide; residual material alone carries the local saturated-core scalar charge. Explicit postulate, not yet derived from a unique action
$[\Pi^r_r]_{r_c^-}^{r_c^+} = 2\tau_{\Sigma}/r_c$; global wall-flux junction open	The local pressure condition is dimensionally homogeneous. Audit A-DSB-1-01 withdraws $\mathcal{B}_{\text{BI}} + 8\pi r_c \tau_{\Sigma} = 0$ because $\mathcal{B}_{\text{BI}}$ as defined has energy units while the second term has force units. The finite-wall action must derive the global junction and its mass scaling; τ_{Σ} may not be fitted
$d(K + E_{12}) = 0$	Demonstrated local mechanical ledger; no local depletion of u_0
Bridge from the local ledger to the universal reservoir CMB and BAO spectrum	Open nonlinear constitutive problem Historical background-protection checkpoint retained; full perturbative derivation for the current HBR/thermal branch remains open

Version history

Version v4.2.1 (1 September 2026 — figure-integrated maintenance revision)

- restoration of the radiative–constitutive checkpoint as Figure 2;
- replacement of the obsolete projector-based Figure 1 by the current CLG-1 chain built from $\mathcal{G}[q]$, $\Phi[q]$, $\Psi_{\mathcal{E}}$, the Poisson limit and the local pair-energy ledger;
- explicit separation in Figure 1 between the demonstrated local conservation law $d(K+E_{12}) = 0$ and the open nonlinear bridge to the universal reservoir;
- no scientific strengthening: all equations, numerical values, rejection conditions and status classifications remain those of v4.2.0.

Version v4.2.0 (1 September 2026 — Pure Cadence and nonlinear Dirac survival audit)

- integration of N-Æ-06, deriving the universal minimal matter interface and the local nonlinear scalar constraint chain of the frozen CNL-2D branch;
- minimal covariant continuation of the constrained antisymmetric sector without a new constant;
- conditional local count of six first-class plus ten second-class constraints, giving $N_{\text{grav}} = 2$ on the regular $k \neq 0$ elliptic branch;
- debt D-Æ-04 split into D-Æ-04A, closed conditionally for the gravitational chain, and D-Æ-04M, open and narrowed to explicit microscopic matter formulations that may modify the canonical structure.

Version v4.1.0 (1 September 2026 — Pure Cadence revision)

- integration of N-Æ-05 and the Pure Cadence Principle;
- conditional fixing of the complete two-derivative cadence dependence $A = P/(4Nc_{\mathcal{E}}^2)$, $B = P/(12Nc_{\mathcal{E}}^2)$, $C = PN$, $D = V = 0$ before the nonlinear Dirac audit;
- conditional closure of D-Æ-03, with D-Æ-04 promoted to the next certification frontier.

Version v4.0.0 (24 August 2026 — conditional PPN closure and minimal two-derivative completion)

- audit correction A-DSB-1-01, incorporated on 26 August 2026 before any recorded public access: withdrawal of the dimensionally inconsistent global surface expression $\mathcal{B}_{\text{BI}}(r_c) + 8\pi r_c \tau_{\Sigma} = 0$ and of its claimed equivalence with the local pressure jump; the local condition remains, while the regulated global junction is returned to the open calculation frontier;

- consolidation of N-Æ-02, extending the same weak-field branch from $\gamma_{\text{PPN}} = 1$ to the conditional tuple $(\gamma, \beta, \alpha_1, \alpha_2) = (1, 1, 0, 0)$ without a PPN fit;
- correction of the apparent $\beta = 0$ branch by separating the physical areal radius from the isotropic PPN radius, giving $C_A = C_\rho - C_\rho^2/2 + O(C_\rho^3)$ and $\beta_{\text{PPN}} = 1$;
- cross-audit against N-Æ-04 V2, adding the closed-form optical index $n_{\min} = (1+u)^3/(1-u)$ and its divergence at the common surface $N = 0$, $C_A = 1$, explicitly conditional on the minimal static branch;
- addition of the universal first-order source coupling and the conditional vector result $\alpha_1 = \alpha_2 = 0$;
- integration of a corrected N-Æ-04: the minimal CNL-2D candidate contains the required $c_{\text{Æ}}^{-2}$ kinetic factor, recovers CLG-1 quadratically and reproduces the static $\beta = 1$ branch;
- derivation of the primary conformal constraint $\pi \approx 0$ at $\lambda = 1/3$ and identification of the elliptic origin of the $k \neq 0$ restriction;
- recognition that the homogeneous kernel permits $\mathcal{K} = 3H(T)$ while leaving $H(T)$ to the independent boundary-production cosmology;
- withdrawal of the claim that CNL-2D is unique: spatial relabelling permits additional functions of the physical cadence field N whose higher Taylor coefficients are invisible at quadratic order;
- registration of D-Æ-03 (cadence minimality) and D-Æ-04 (complete nonlinear Dirac matrix with microscopic matter) as the two origin-protection debts;
- explicit restriction of the PPN claim to the four parameters calculated; no assertion is made for the remaining PPN coefficients.

Version v3.11.0 (22 August 2026 — the optical operator of State Æ and the Cassini closure)

- integration of Note N-Æ-02, answering the CASSINI CALCULATION CONTRACT of Section XII without a new field, a new length scale or an adjustable optical coefficient;
- Lemma of eikonal algebraicity: in the geometric-optics limit only the algebraic dependence on the relational state enters the principal symbol, which closes the gradient-constitutive branch before the question of fitting arises;
- Lemma of labelling-gauge rigidity: the unique relabelling-covariant electromagnetic coupling is $-\frac{1}{2}q_{ij}\Sigma^{ij}$ with coefficient exactly one, verified symbolically; any other value promotes the labels to observables and degrades the first-class constraints of CLG-1 §VII;
- derivation of the principal eikonal symbol $\mathfrak{P} = \omega^2/(N^2 c_{\text{Æ}}^2) - g^{ij}[q]k_i k_j$ on $\Omega_{\text{Æ}}(T)$ with absolute time and no fourth physical dimension, the four-index object being the symbol of a medium and not a manifold;

- derivation of $n^2 = (1 + \psi)/(1 - C)$ and of $\gamma_{\text{PPN}} = \psi/C = -\text{sgn}(\sigma)$, hence the quantisation $\gamma_{\text{PPN}} \in \{-1, +1\}$: RM contains no continuous optical parameter;
- audit A-Æ-02: the v3.10.1 exclusion of $\delta_{ij} + q_{ij}$ was conditional on $\sigma > 0$, a sign fixed nowhere in v3.10.1, the pure relational sector being invariant under the simultaneous reversal of q_{ij} , B_i , Λ and σ ;
- selection of $\text{sgn}(\sigma) = -1$ by internal consistency with the DSB-1 optical threshold and by the qualitative sign of the solar time delay, giving $\gamma_{\text{PPN}} = +1$ exactly, 0.91σ from the Cassini central value and a solar-limb deflection of $1.75119''$;
- verification that every previously derived result — the Poisson limit, E_{12} , u_{rel} , Π_{ij} , the Dirac count, ε_M — is invariant under the flip, only the reported value of ψ reversing;
- registration of debt D-Æ-02: the spatial labels cannot be both a gauge redundancy and physically individuated, and the arbitration conditions the optical closure and the degree count alike;
- explicit refusal of any distinctness claim at this order: the derived symbol is algebraically identical to the linearised general-relativistic operator in isotropic gauge, and the closure is recorded as survival at the frontier, not as an advance beyond it;
- transfer of absolute priority to β_{PPN} , then to the preferred-frame parameters α_1 and α_2 , to be derived on the same action with the same Z , $|\sigma|$ and $\sigma < 0$;
- preservation, without alteration, of every v3.10.1 statement, register row, rejection condition and frontier item, including those the present edition supersedes.

Version v3.10.1 (22 August 2026 — non-saturated stellar state and Cassini frontier)

- integration of Note N-Æ-01 as a parameter-free isotropic reduction of the already retained CLG-1 branch;
- derivation of $\psi = \Phi[q]$ and, under the stated weak-field normalisation, $\psi = -\text{sgn}(\sigma)C$;
- derivation of the static relational field energy $u_{\text{rel}} = -|\nabla\Psi_{\text{Æ}}|^2/(8\pi G)$ and of the inverse-Maxwell stress balance $\partial_j(P\delta_{ij} - \Pi_{ij}) = 0$;
- append-only correction C-Æ-01: the relational stress is compressive along the field and tensile transversely, reversing the former verbal assignment;
- solar scale audit showing $X_{\mathcal{R}} \simeq 7.1 \times 10^{-181}$ on the retained Planck-length branch; the former ordinary-stellar rupture reading is withdrawn;
- restriction of the achromaticity statement to a one-way conditional implication requiring Dirac closure, absence of additional dissipative channels, all-spectrum losslessness and the appropriate causal high-frequency assumptions;

- isolation of the Cassini factor-of-two problem as an anomaly internal to RM: the weak temporal coefficient is retained, the spatial optical coefficient is open, and the naive identification of q_{ij} with an optical metric is explicitly forbidden;
- adoption of a no-fit Cassini calculation contract: the electromagnetic principal eikonal symbol must derive γ_{PPN} from the admissible relational variables or reject the local branch.

Version v3.10.0 (21 August 2026 — optical–mechanical double-scale revision)

- withdrawal of the progressive-conversion reading of Zone II: the region $r_c < r < R_S$ remains State $\mathcal{A}$ and carries an increasing nonlinear relational stress;
- reclassification of $R_S \propto M$ as the optical and transport threshold, while $X_{\mathcal{R}}(r_c) = 1$ defines the mechanical $\mathcal{A} \rightarrow E$ boundary;
- recognition that $r_c \propto M^{1/3}$ implies the universal ratio $M/r_c^3 = c_{\mathcal{A}}^2/(\sqrt{6}G\ell_\star^2)$ and therefore supplies the scaling signature of a three-dimensional incompressible E volume;
- exact evaluation of the Born–Infeld shell integrals $(\pi - 3)/6$ for the bulk energy and $(5 - \pi)/6$ for the magnitude of the algebraic scalar-stress projection;
- derivation of the local isotropic metric variation Π^i_i , including the double-divergence terms that become a boundary flux at r_c ;
- historical replacement of the sole pressure-jump target by the proposed surface equation $\mathcal{B}_{\text{BI}}(r_c) + 8\pi r_c \tau_\Sigma = 0$; this expression is withdrawn by audit A-DSB-1-01 in the corrected v4.0.0 file because it is dimensionally inconsistent;
- withdrawal of the identification of Hawking radiation with the latent heat of the mechanical transition at R_S . The exact temperature identity at R_S is retained as an optical-surface consistency result; its transport mechanism remains open;
- preservation of CLG-1, CNL-BI1, OCH-1, the residual fraction branch and every v3.9.2.1 weak-field result not affected by this reinterpretation.

Version v3.9.2 (19 August 2026 — ontological source-separation freeze)

- identification of the apparent two-mass tension: the exterior curvature scale uses the complete macroscopic mass M , whereas the saturated-core scalar constraint uses the residual material density built from $\varepsilon_M M$;
- adoption of OCH-1, $M_{\text{ext}} = M$ and $\rho_{\text{rel}}^{(\text{core})} = \rho_{m,\text{res}}$, as an explicit constitutive source-selection hypothesis;
- preservation of the cohesive and conversion sectors in the global mass, energy and radial-stress ledger; OCH-1 does not declare state E or Zone II massless;
- three rejection tests: equal multiplier charge for E and residual matter, an exterior field sourced only by $\varepsilon_M M$, or failure of the radial ledger without a new fitted coupling.

Version v3.9.1 (19 August 2026 — residual-admissibility freeze)

- derivation of the elastic-normalization bound $0 < Z \leq c_{\mathcal{E}}^4/(4\pi G)$ from the physical requirement $0 \leq \varepsilon_M \leq 1$;
- isolation of the independently normalized weak-field branch $|\sigma| = 16\pi G/c_{\mathcal{E}}^2$, for which $Z = c_{\mathcal{E}}^4/(32\pi G)$ and $\varepsilon_M = \sqrt{2}/4 \simeq 0.353553$;
- explicit prohibition on using the numerical residual fraction to select σ : the implication is a prediction conditional on the normalization, not a reverse calibration.

Version v3.9.0 (19 August 2026 — double-scale Black-Core candidate edition)

- reinterpretation of the M versus $M^{1/3}$ scaling difference as two distinct physical thresholds rather than one failed radius;
- freezing of $R_S = 2GM/c_{\mathcal{E}}^2$ as the exterior thermodynamic conversion surface and $r_c = 6^{1/6}(\ell_\star^2 GM/c_{\mathcal{E}}^2)^{1/3}$ as the inner curvature barrier;
- definition of the three zones: transmissive exterior, conversion layer, and regular E -rich core carrying residual material memory;
- derivation of the Hawking-temperature identity at R_S , explicitly separated from the central regularisation mechanism;
- derivation of $\rho_{\text{res}} = 3\varepsilon_M c_{\mathcal{E}}^2/(4\pi\sqrt{6}G\ell_\star^2)$, in which the total mass cancels exactly;
- conditional determination $\varepsilon_M = 4\pi\sqrt{2}G/(c_{\mathcal{E}}^2|\sigma|) = 2\sqrt{\pi G Z}/c_{\mathcal{E}}^2$ from the isotropic saturation limit and the existing matching $Z\sigma^2 = 8\pi G$;
- demonstration of a finite central constrained limit, $g(r) \propto r$ and $g(0) = 0$, for the uniform residual-core reduction;
- isolation of the final radial-pressure lock $[\Pi^r_r] = 2\tau_\Sigma/r_c$, with τ_Σ derived from the phase-field energy rather than admitted as a fitting parameter;
- explicit falsification rule: if the pressure junction does not return the frozen r_c or requires a mass-dependent residual fraction, DSB-1 is rejected, not recalibrated.

Version v3.8.0 (19 August 2026 — gauge-invariant nonlinear candidate edition)

- introduction of CNL-BI1, the first nonlinear saturation candidate built exclusively from derivatives of spatial-gauge covariant relational quantities;
- definition of the nonlinear relational tensor γ_{ij} without a fourth physical dimension and without identifying it with a deformable physical space;
- derivation of $\mathcal{R}^{(1)}[\gamma] = -\mathcal{G}[q]$ and identification of the independent three-dimensional curvature invariants;

- definition of the minimal positive norm $X_{\mathcal{R}} = \ell_{\star}^4 \mathcal{R}_{ij} \mathcal{R}^{ij}$, which retains both scalar and trace-free tidal curvature;
- introduction of the subtracted Born–Infeld density $u_{\star}[1 - \sqrt{1 - X_{\mathcal{R}}} - X_{\mathcal{R}}/2]$. The subtraction removes the entire $O(X_{\mathcal{R}})$ term, so the correction begins at $O(q^4)$ and cannot alter CLG-1 at quadratic order;
- derivation of the finite-density, divergent-response limit at $X_{\mathcal{R}} = 1$ and of the weak spherical scale $r_{\text{sat}} \propto M^{1/3}$;
- explicit refusal to identify this local curvature scale with the Black-Core phase boundary: regular spherical solutions, the nonlinear Dirac algebra, ultraviolet stability and the global phase-matching law remain open;
- restriction of the older $P(X)/U(\Gamma)$ reduction to the homogeneous cosmological auxiliary sector. It is not admissible as the local gauge-invariant completion.

Version v3.7.0 (19 August 2026 — local gravitational closure edition)

- replacement of the former projector $(1 - \mathcal{P}_0) \text{Tr } Q$ and the independent $\delta\Gamma$ action by CLG-1, a local quadratic candidate action written only on $(T, \mathbf{x})$;
- redefinition of Q_{ij} as a relational state tensor, not a deformation of space or of the homogeneous fundamental density u_0 ;
- introduction of the spatial gauge invariant $\mathcal{G}[q] = \nabla^2 \text{Tr } q - \partial_i \partial_j q_{ij}$, which annihilates the homogeneous mode without an imposed global projector;
- derivation, for $k \neq 0$, of $\nabla^2 \Psi_{\mathcal{E}} = 4\pi G \rho_m$ and $E_{12} = -G m_1 m_2 / r$ from the single matching $Z\sigma^2 = 8\pi G$;
- certification of the Dirac count with six first-class and ten second-class constraints. The corrected determinant is $\det \Omega_{\text{SC}} = \frac{B^2 \mu_A^6}{16} (k^2)^{10}$, leaving two local tensor polarisations when $B\mu_A k \neq 0$;
- withdrawal of the local ledger $d_{\text{grav}} K = -V_{\mathcal{E}} d_{\text{grav}} u_0$. The demonstrated statement is $d(K + E_{12}) = 0$; the bridge to a universal reservoir remains open;
- exclusion of the former $\mathcal{F}_{\text{sat}}(Q)$ from the local gravitational action because its raw trace and symmetric terms break the adopted spatial gauge. Nonlinear saturation is retained as an open programme, not silently repaired;
- explicit limits: no claim for $k = 0$, nonlinear closure, PPN parameters, strong gravity, point-source self-energies, ultraviolet stability or the microscopic matter coupling.

Version v3.6.5 (18 August 2026 — typographic pass)

- no scientific content altered; this edition changes layout only;

- A spacing rule was declared for chapter titles. The document loads `titlesec` with a title format but no spacing rule, so the package had applied its default vertical skip above every chapter title. Titles now begin at the top of their page; the edition is four pages shorter as a result;
- the table of contents is moved behind the preface and the epistemic contract. A reader opening the document now meets the preface first, then the contract, then the navigation apparatus.

Version v3.6.4 (18 August 2026 — epistemic contract edition)

- no result, equation or numerical value removed or altered; the frozen chain PF-e-R1 and $R_{NC}^{\text{anc}}(A) = 262\,211.6$ ly stand unchanged;
- addition of the front-matter chapter *What This Document Establishes, What It Postulates, and What Would Refute It*, placed between the preface and the reading guide. The preface states why the work exists; the contract states what it is worth;
- the contract records explicitly that the medium action, written in full, propagates nine modes, and that reducing this to two requires three independent constitutive clauses — gauge invariance of the labelling, absence of a scalar kinetic term, absence of an antisymmetric kinetic term — under which the resulting mode structure is exactly that of linearised general relativity;
- consequent statement of the standing debt: RM must presently impose what general relativity derives. The clauses are declared and dated rather than concealed;
- registration of two constitutive defects to be arbitrated: the non-convexity of $\mathcal{F}_{\text{sat}}$ in the antisymmetric sector, requiring α and β of opposite signs; and the ambiguity of the medium-action kinetic index, to be resolved by the $3 + 1$ form with one coefficient per sector;
- registration of two new observational exposures opened by that rewriting: the tensor speed $v^2 = \mu_0/Z_T$ against the GW170817 bound, and the tensor mass $m^2 = 2C\alpha/Z_T$ against gravitational-wave dispersion bounds;
- registration of the self-imposed bound: maximal power extractable from the $\mathbb{A}$ gravitomagnetic channel of order 5×10^2 W for the Earth, derived from $\tau_0 > 10^{17}$ s;
- correction of an undefined control sequence `\anc` introduced in v3.6.3, which prevented clean compilation.

No Absolute Principle was modified. No conditional concordance was promoted to a prediction.

Version v3.6.3 (17 August 2026 — status requalification edition)

- no result, equation or numerical value removed; this edition corrects announced status only;

- Chapter 9: the claim that the information paradox is solved is withdrawn. The chapter now states explicitly that (9.1)–(9.4) reproduce the 1974 Bekenstein–Hawking integration, that the RM content lies in the identification $C = -2\Psi_{\mathbb{E}}/c^2$, and that an area-proportional entropy is part of the statement of the unitarity problem rather than its answer;
- the four entries of the v3.6.2 revision record now carry explicit T0–T3 status labels, consistent with the discipline applied to PF-e-R1 elsewhere in the document;
- founding statement on entropy and its constitutional-table row requalified to T1, consistency result;
- lexical sweep of “solved”, “exact derivation”, “analytically derived”, “proved”;
- PF-e-R1 and its frozen chain are untouched: $R_{NC}^{\text{anc}}(A) = 262\,211.6$ ly stands unchanged.

No Absolute Principle was modified. No conditional concordance was promoted to a prediction.

Version v3.6.2 (17 August 2026 — The Absolute Audited Closure Edition)

- restauration de l’audit de consolidation rigoureux (v3.6.1) séparant strictement la géométrie locale des trous noirs de la topologie globale du Noyau Cosmique ;
- démonstration mathématique de la dégéométrisation locale : la métrique optique $g_{00} = 1 - C(r)$ est dérivée du champ scalaire, remplaçant la singularité de Schwarzschild par le seuil de saturation absolu $C = 1$;
- dérivation thermodynamique de l’Entropie de Bekenstein-Hawking ($S \propto A$) comme la mesure exacte de l’information structurelle ("Mémoire Cosmique") cristallisée à la frontière de phase $C = 1$;
- preuve du Théorème de Protection Primordiale garantissant l’annulation asymptotique de l’énergie élastique par rapport à la densité de matière au temps du CMB ($z \rightarrow 1100$) ;
- épuration formelle de l’historique des versions pour clôturer le manifeste.

The Optical Operator of State $\mathcal{A}$ — N- $\mathcal{A}$ -02

STATUS OF THIS NOTE

This consolidated note answers the CASSINI CALCULATION CONTRACT and extends the same weak-field branch to the remaining principal PPN locks. The optical operator still gives $\gamma_{\text{PPN}} = +1$ on the internally selected branch $\sigma < 0$, with no adjustable optical coefficient. A universal first-order matter coupling, conditional on the three-dimensional material momentum balance, preserves the six first-class constraints and gives $\alpha_1 = \alpha_2 = 0$. The apparent alternative $\beta_{\text{PPN}} = 0$ is withdrawn: it resulted from comparing the exact cadence law written with the physical areal radius to the PPN series written with the isotropic radius. Their explicit conversion gives $\beta_{\text{PPN}} = 1$. The same result follows from the minimal constant-coefficient two-derivative completion CNL-2D. The numerical values are therefore closed conditionally and without fitting. What remains open is the protection of their origin: the complete microscopic matter action, the nonlinear Dirac bracket matrix, and a derivation excluding additional admissible functions of the physical cadence field N .

I. The admissible ingredients

The contract permits T , $\Omega_{\mathcal{A}}(T)$ and the admissible relational invariants; it forbids a fourth physical dimension and any optical parameter adjusted to force the answer. The inventory available in the static exterior of an isolated star is therefore:

1. absolute time T , with ω its conjugate in the eikonal ansatz;
2. the domain $\Omega_{\mathcal{A}}(T)$ carrying flat labels x^i and the labelling gauge of CLG-1 §II,

$$\delta q_{ij} = \partial_i \xi_j + \partial_j \xi_i, \quad \delta B_i = \partial_T \xi_i; \quad (10.5)$$

3. the relational state tensor q_{ij} , whose isotropic representative satisfies, by Lemma 1 and Lemma 2 of N- $\mathcal{A}$ -01,

$$q_{ij} = \psi \delta_{ij}, \quad \psi = \Phi[q] = -\text{sgn}(\sigma) C(r); \quad (10.6)$$

4. the already registered cadence coefficient, now written with the physical areal radius r_A ,

$$N^2(r_A) \equiv g_{00}^{\text{eff}} = 1 - C_A(r_A), \quad C_A = \frac{2GM}{r_A c_{\mathcal{A}}^2} = -\frac{2\Psi_{\mathcal{A}}}{c_{\mathcal{A}}^2}; \quad (10.7)$$

at first order $C_A = C_\rho + O(C_\rho^2)$, where $C_\rho = 2GM/(\rho c_{\mathcal{A}}^2)$ is the compactness written with the isotropic PPN radius ρ ;

5. the electromagnetic field written without a fourth component: E_i and B^i on $\Omega_{\mathbb{E}}(T)$, with $E_i = -\partial_T A_i - \partial_i A_0$ and $\sqrt{g} B^i = \epsilon^{ijk} \partial_j A_k$, where A_0 enters only as the multiplier of the Gauss constraint.

Two objects that might have been expected are absent and must be recorded as absent. The local gauge-invariant scalar $\mathcal{G}[q] = \sigma \rho_m$ *vanishes identically in the exterior vacuum*, and the antisymmetric sector a_{ij} and the velocity invariant K_{ij}^{TF} vanish in a static configuration. Consequently no local gauge invariant of the relational sector is available at first order in the region traversed by the Cassini radio link.

II. Lemma 1 — eikonal algebraicity

Statement. In the geometric-optics limit, the principal symbol can depend on the relational state only *algebraically*. Any constitutive term containing derivatives of the state, such as $\ell^2 \partial_i \partial_j C$, is suppressed by at least one power of $(kL)^{-1}$, where L is the field gradient scale, and cannot appear in the principal part.

Proof. Write the constitutive tensors as $\varepsilon_{ij} = \varepsilon_{ij}(q, \ell \partial q, \dots)$. Under the eikonal ansatz $A_i \sim a_i e^{iS/\epsilon}$ with $k_i = \partial_i S/\epsilon$, each explicit derivative acting on the slowly varying state contributes $O(\epsilon^0)$ while each derivative acting on the phase contributes $O(\epsilon^{-1})$. The principal symbol collects the terms of order ϵ^{-2} ; only the algebraic dependence survives. $\square$

Consequence. Any gradient-type constitutive term would in addition require a new length scale ℓ , which the contract forbids as a fitted optical parameter. The lemma removes that branch before the question of fitting arises. The optical operator is therefore controlled by q_{ij} and N evaluated pointwise.

III. Lemma 2 — labelling-gauge rigidity

Statement. If the electromagnetic sector respects Eq. (10.5), its coupling to the relational state is unique: indices are contracted with

$$g_{ij} \equiv \delta_{ij} + q_{ij}, \quad g^{ij} = \delta^{ij} - q^{ij} + O(q^2), \quad \sqrt{g} = 1 + \frac{1}{2} \text{Tr } q + O(q^2), \quad (10.8)$$

with coefficient exactly one. No relative coefficient $\lambda \neq 1$ is admissible.

Proof. Expand the most general quadratic, relabelling-covariant electromagnetic action built with $\delta_{ij} + \lambda q_{ij}$ to first order in q . Direct computation (verified symbolically) gives

$$S = S_0[A] - \frac{\lambda}{2} \int dT d^3x q_{ij} \Sigma^{ij}, \quad \Sigma^{ij} = E^i E^j + B^i B^j - \frac{1}{2} \delta^{ij} (E^2 + B^2), \quad (10.9)$$

where Σ^{ij} is the Maxwell stress tensor. Under Eq. (10.5) the coupling term varies by $+\lambda \int \xi_j \partial_i \Sigma^{ij}$ after integration by parts, while the variation of S_0 under the Lie transport $\delta A_i = \mathcal{L}_\xi A_i$ produces exactly $-\int \xi_j \partial_i \Sigma^{ij}$. The two cancel if and only if $\lambda = 1$. $\square$

WHY THIS IS NOT A FITTED PARAMETER

The number $\lambda = 1$ is not chosen, measured or matched. It is the unique value for which the statement of CLG-1 §II — *the labels assigned to a local configuration carry no independent physics* — remains true once light is present. Any other value promotes the labels to observables and, by the same token, converts the first-class constraints of CLG-1 §VII into second-class ones, resurrecting the modes that the Dirac count removed. Fixing λ is a survival condition of the degree count $N_{\text{phys}} = 2$, not an optical adjustment.

IV. Lemma 3 — cadence universality

Statement. The powers of N entering the electromagnetic action are fixed by the Exclusion Interaction (historical PS-0 label): an observer using local proper units measures the saturated speed $c_{\mathcal{E}}$, so the local electromagnetic Lagrangian is the ordinary one written in proper time $d\tau = N dT$ and proper volume $\sqrt{g} d^3x$. This gives

$$S_{\text{EM}} = \frac{1}{2} \int dT d^3x \sqrt{g} \left[\frac{g^{ij} E_i E_j}{N c_{\mathcal{E}}^2} - N g_{ij} B^i B^j \right]. \quad (10.10)$$

No other distribution of N is compatible with the statement that the local branch is saturated at $c_{\mathcal{E}}$.

Status. This is a constitutive statement, not a consequence of the CLG-1 action, which contains no lapse. It is the RM form of local position invariance: the same cadence factor governs material clocks and the electromagnetic operator. It is recorded as such in Section XI.

V. Theorem 1 — the principal eikonal symbol

Insert $A_i = a_i \exp[i(k_j x^j - \omega T)]$ into Eq. (10.10), hold N and g_{ij} fixed at the eikonal order permitted by Lemma 1, and vary. For the transverse polarisations the symbol collapses to a single scalar condition.

THE PRINCIPAL SYMBOL OF THE ELECTROMAGNETIC OPERATOR ON STATE $\mathcal{E}$

$$\mathfrak{P}(\omega, k; x) = \frac{\omega^2}{N^2(x) c_{\mathcal{E}}^2} - g^{ij}[q](x) k_i k_j = 0$$

with $N^2 = 1 - C$ from Eq. (10.7) and g^{ij} from Eq. (10.8). Only T , $\Omega_{\mathcal{E}}(T)$, the relational state and the already registered cadence enter. The four-index appearance of $\mathfrak{P}$ is a property of the *symbol* of a hyperbolic operator in $3 + 1$ absolute-time form; it is the optical structure of a medium, not a fourth physical dimension.

Explicit coefficients. For the isotropic representative $g_{ij} = (1 + \psi)\delta_{ij}$ one has $\sqrt{g} = (1 + \psi)^{3/2}$, and the two coefficients of Eq. (10.10) read

$$\text{coefficient of } \omega^2 : \quad \frac{(1 + \psi)^{1/2}}{N c_{\mathcal{A}}^2}, \quad \text{coefficient of } k^2 : \quad N (1 + \psi)^{-1/2}. \quad (10.11)$$

Equivalently, State $\mathcal{A}$ behaves as a medium with

$$\varepsilon^{ij} = \mu^{ij} = \frac{\sqrt{g} g^{ij}}{N}. \quad (10.12)$$

VI. Weak-field reading of the two optical coefficients

Setting $\mathfrak{P} = 0$ gives the exact phase velocity and index in this representative,

$$v^2 = \frac{N^2 c_{\mathcal{A}}^2}{1 + \psi} = \frac{c_{\mathcal{A}}^2 (1 - C)}{1 + \psi}, \quad \boxed{n^2 = \frac{1 + \psi}{1 - C}}. \quad (10.13)$$

Expanded to first order in the solar field,

$$\underbrace{\frac{1}{N^2} = 1 + C + O(C^2)}_{\text{temporal optical coefficient}}, \quad \underbrace{g^{ij} = (1 - \psi)\delta^{ij} + O(\psi^2)}_{\text{spatial optical coefficient}}, \quad n = 1 + \frac{C + \psi}{2} + O(2). \quad (10.14)$$

The standard PPN light-propagation form is $n = 1 + \frac{1}{2}(1 + \gamma_{\text{PPN}})C$. Comparison is immediate.

THEOREM 2 — THE PPN COEFFICIENT OF RM

$$\boxed{\gamma_{\text{PPN}} = \frac{\psi}{C} = -\text{sgn}(\sigma)}$$

γ_{PPN} is the ratio of the isotropic relational representative to the cohesion fraction. Because Eq. (10.6) forces $|\psi| = |C|$ on the weak branch, RM admits *no continuum of values*: $\gamma_{\text{PPN}} \in \{-1, +1\}$.

The two branches, explicitly.

Branch	ψ	exact index $n(C)$	γ_{PPN}
$\sigma > 0$	$-C$	$n \equiv 1$ for all C	-1
$\sigma < 0$	$+C$	$n = \sqrt{(1 + C)/(1 - C)}$	$+1$

VII. Audit finding A-Æ-02

AUDIT A-Æ-02 — THE SECTION XII EXCLUSION WAS CONDITIONAL

Section XII of v3.10.1 excludes the identification $\gamma_{ij} = \delta_{ij} + q_{ij}$ on the ground that it gives $(1 - C)\delta_{ij}$ and hence the wrong first-order spatial sign. That computation is correct, but it is prefaced in the source text by the words *for $\sigma > 0$* , and the same restriction is carried explicitly by register entry N-Æ-01.2 and by Eq. (10.6), which retains $\text{sgn}(\sigma)$ throughout. Searching v3.10.1 exhaustively: the matching $Z\sigma^2 = 8\pi G$ fixes σ^2 ; the weak-field normalisation fixes $|\sigma| = 16\pi G/c_{\text{Æ}}^2$; the observable potential $\Psi_{\text{Æ}} = Z\sigma\Phi \propto \sigma^2$, the relational energy u_{rel} , the stress Π_{ij} , the pair energy E_{12} , the Dirac count and ε_M all depend on σ only through σ^2 or $|\sigma|$. **No statement in v3.10.1 fixes the sign of σ .** The exclusion in Section XII therefore disposed of one branch and left the other unexamined. This finding is appended; the Section XII wording is not rewritten.

Structural reason for the ambiguity. The pure relational sector possesses the exact discrete redundancy

$$(q_{ij}, B_i, \Lambda, \sigma) \longrightarrow (-q_{ij}, -B_i, -\Lambda, -\sigma), \quad (10.15)$$

under which every term of the CLG-1 action is invariant: $K_{ij}^{\text{TF}} K_{\text{TF}}^{ij}$ and $\mathcal{V}_{\text{FP}}^{(3)}[q]$ are quadratic, and $\Lambda(\mathcal{G}[q] - \sigma\rho_m)$ has both factors reversed. The sign of σ is thus *unobservable in the gravitational sector alone*. Lemma 2 breaks the redundancy, because δ_{ij} in $g_{ij} = \delta_{ij} + q_{ij}$ is inert under Eq. (10.15). *The sign of σ becomes physical exactly when light is coupled, and not before.* This is why it could not have been fixed anywhere in v3.10.1, and why it is fixed here.

VIII. Internal selection of the branch

The selection is made without reference to the Cassini number.

Argument 1 — DSB-1 requires an optical threshold. DSB-1 registers R_S as *the optical and transport threshold*, with $C(R_S) = 1$, Zone I transmissive, and the Hawking identity $T_H = \hbar c_{\text{Æ}} |\nabla C(R_S)| / (4\pi k_B)$ attached to that surface. On the branch $\sigma > 0$, Eq. (10.13) gives $n^2 = (1 - C)/(1 - C) = 1$ *identically, at every field strength*: State Æ is optically trivial, there is no optical threshold anywhere, and Zone I, the transport boundary and the optical reading of the Hawking identity all lose their object. On the branch $\sigma < 0$, $n^2 = (1 + C)/(1 - C)$ diverges precisely at $C = 1$. Only $\sigma < 0$ is consistent with the architecture already registered in DSB-1.

Argument 2 — the sign of the Shapiro delay. On $\sigma > 0$ the index is exactly unity, so the solar time delay and the deflection are exactly zero, not merely halved. This branch is excluded by any single qualitative observation of solar light bending or radio delay, at 1919 or 1964 precision. No modern datum is needed.

SELECTOR

$\text{sgn}(\sigma) = -1$, hence $\psi = +C$ and $\gamma_{\text{PPN}} = +1$ exactly. What is fixed is one discrete bit, already present in the theory and previously unobservable. It is not a continuous optical coefficient, and it cannot be tuned: the magnitude $|\gamma_{\text{PPN}}| = 1$ is rigid on both branches.

IX. Confrontation

With $\gamma_{\text{PPN}} = 1$ exactly, RM predicts $\gamma_{\text{PPN}} - 1 = 0$ with no error bar of its own.

Observable	$\sigma < 0$ (RM)	$\sigma > 0$	measured
$\gamma_{\text{PPN}} - 1$	0 exactly	-2	$(2.1 \pm 2.3) \times 10^{-5}$
Solar-limb deflection	1.75119''	0	$\simeq 1.75''$
Shapiro delay, Cassini geometry	262 μs	0	non-zero, measured
Optical threshold at $C = 1$	present	absent	required by DSB-1

The RM prediction sits at 0.91σ from the Cassini central value ($p \simeq 0.36$): a null result, which is the correct outcome for a parameter-free prediction. Note that the previously feared “Newtonian half”, $\gamma_{\text{PPN}} = 0$, is *not* an available RM branch at all; had it been, it would already have been excluded in 1915 by the perihelion of Mercury, which it underpredicts by a factor of three (14.3'' against 42.98'' per century).

X. Consistency of $\sigma < 0$ with the rest of Pillar II

Every previously derived result is verified to be invariant under the flip:

1. $\nabla^2 \Psi_{\mathcal{E}} = 4\pi G \rho_m$: $\Psi_{\mathcal{E}} = Z\sigma\Phi$ with $\Phi \propto \sigma$, hence $\Psi_{\mathcal{E}} \propto \sigma^2$. Unchanged, with $\Psi_{\mathcal{E}} < 0$ and $C > 0$ as before.
2. $E_{12} = -Gm_1m_2/r$: depends on $Z\sigma^2$. Unchanged, attraction preserved.
3. $u_{\text{rel}} = -|\nabla \Psi_{\mathcal{E}}|^2/(8\pi G)$ and Π_{ij} : functions of $\Psi_{\mathcal{E}}$ only. Unchanged, including correction C-Æ-01.
4. Dirac count $N_{\text{phys}} = 2$, $\det \Omega_{\text{SC}} = B^2 \mu_A^6 (k^2)^{10}/16$: no σ dependence. Unchanged.
5. $\varepsilon_M = 2\sqrt{\pi G Z}/c_{\mathcal{E}}^2 = \sqrt{2}/4$ and $Z = c_{\mathcal{E}}^4/(32\pi G)$: functions of Z and $|\sigma|$. Unchanged.
6. Only the reported value of the isotropic representative reverses: $\psi = +C$ in place of $\psi = -C$. Register entry N-Æ-01.2, which is explicitly conditional on $\sigma > 0$, is not corrected but superseded on the selected branch, append-only.

Physical reading permitted by RM. With $\psi = +C$, the relational state carries a positive isotropic representative near a mass. The continuum is not stretched and u_0 is untouched; what

increases is the relational separation that State $\mathcal{A}$ interposes between neighbouring labels. Light, propagating on the saturated branch, must traverse more relational steps per label interval: hence delay and bending. The factor of two of the Cassini frontier is the sum of two equal contributions of the *same* field — one in the cadence coefficient $1/N^2 = 1 + C$, one in the spatial coefficient $g^{ij} = (1 - \psi)\delta^{ij}$ — and not a doubling of a single effect.

XI. Universal matter current and the preferred-frame parameters

The same relabelling symmetry that fixed the optical coupling also fixes the relative coupling of the auxiliary field B_i to material momentum. Let j^i denote the three-dimensional material momentum density and Σ^{ij} its spatial flux. The most general first-order source term with independent coefficients is

$$S_{j\Sigma}^{(1)} = - \int dT d^3x \left(g_B B_i j^i + \frac{g_Q}{2} q_{ij} \Sigma^{ij} \right). \quad (10.16)$$

Under Eq. (10.5), integration by parts gives

$$\delta S_{j\Sigma}^{(1)} = \int dT d^3x \xi_i \left(g_B \partial_T j^i + g_Q \partial_j \Sigma^{ji} \right). \quad (10.17)$$

The material equation

$$\partial_T j^i + \partial_j \Sigma^{ji} = 0 \quad (10.18)$$

therefore makes the source invariant if and only if $g_B = g_Q$. Universal minimal coupling to the same relational labels $g_{ij} = \delta_{ij} + q_{ij}$ sets their common normalisation to one:

$$\boxed{g_B = g_Q = 1.} \quad (10.19)$$

The equality is a consequence of the gauge and Eq. (10.18); the unit value is the explicit universal minimal-coupling clause. A complete microscopic action for matter must still derive this clause rather than merely reproduce its first variation.

The gravitational momentum remains

$$\pi^{ij} = \frac{Z}{c_{\mathcal{A}}^2} K_{\text{TF}}^{ij}, \quad p_{B_i} \approx 0. \quad (10.20)$$

Preservation of p_{B_i} gives the sourced momentum constraint

$$\boxed{\mathcal{H}_i^{\text{tot}} = -2\partial_j \pi^{ji} + j_i \approx 0.} \quad (10.21)$$

For a dynamical source transforming with the labels, or for an external source satisfying Eq. (10.18), the generator

$$G[\xi] = \int d^3x \left(\dot{\xi}_i p_B^i + \xi_i \mathcal{H}_i^{\text{tot}} \right) \quad (10.22)$$

continues to generate the spatial relabelling. At weak-source order the six constraints $(p_{B_i}, \mathcal{H}_i^{\text{tot}})$ remain first class. The source term is linear in B_i and q_{ij} and introduces no new gravitational

velocity; consequently the registered second-class block remains

$$\det \Omega_{\text{SC}} = \frac{B^2 \mu_A^6}{16} (k^2)^{10} \quad (10.23)$$

for $B\mu_A k \neq 0$. Matter carries its own physical degrees of freedom, but no additional gravitational polarisation is generated at this order.

For a stationary weak current define $b_i = B_i/c_{\mathcal{E}}$. Variation with respect to B_i gives

$$\left(\nabla^2 \delta_{ij} + \frac{1}{3} \partial_i \partial_j \right) b_j = -\frac{c_{\mathcal{E}}}{2Z} j_i = -\frac{16\pi G}{c_{\mathcal{E}}^3} j_i, \quad (10.24)$$

where the last equality uses $Z = c_{\mathcal{E}}^4/(32\pi G)$. Introduce the dimensionless PPN vector potential by

$$\nabla^2 V_i = -\frac{4\pi G}{c_{\mathcal{E}}^3} j_i. \quad (10.25)$$

Resolving Eq. (10.24) into transverse and longitudinal projectors gives

$$b_i = 4 \left(\mathbb{P}_T + \frac{3}{4} \mathbb{P}_L \right) V_i. \quad (10.26)$$

Using

$$\mathbb{P}_T V_i = \frac{1}{2} (V_i + W_i), \quad \mathbb{P}_L V_i = \frac{1}{2} (V_i - W_i), \quad (10.27)$$

one obtains

$$b_i = \frac{7}{2} V_i + \frac{1}{2} W_i. \quad (10.28)$$

On the conservative weak branch with $g_{0i}^{\text{eff}} = -b_i$ and $\gamma_{\text{PPN}} = 1$, the standard preferred-frame template reads

$$g_{0i}^{\text{eff}} = -\frac{1}{2} (7 + \alpha_1 - \alpha_2) V_i - \frac{1}{2} (1 + \alpha_2) W_i. \quad (10.29)$$

Comparison with Eq. (10.28) yields

THE PREFERRED-FRAME COEFFICIENTS

$$\alpha_1 = 0, \quad \alpha_2 = 0.$$

These are conditional predictions of the universal minimal matter coupling, not fitted cancellations.

XII. The double-radius closure and β_{PPN}

The earlier inference $N^2 = 1 - C \Rightarrow \beta_{\text{PPN}} = 0$ compared two different radial conventions. The exact compactness law uses the physical areal radius r_A , whereas the PPN series uses the isotropic radius ρ . This distinction is sufficient to remove the apparent contradiction.

On the selected branch $\gamma_{\text{PPN}} = 1$, the first-order isotropic spatial operator gives

$$r_A = \rho \left(1 + \frac{1}{2} C_\rho + O(C_\rho^2) \right), \quad C_\rho = \frac{2GM}{\rho c_{\mathbb{E}}^2}. \quad (10.30)$$

Therefore

$$C_A = \frac{2GM}{r_A c_{\mathbb{E}}^2} = C_\rho - \frac{1}{2} C_\rho^2 + O(C_\rho^3). \quad (10.31)$$

Inserting Eq. (10.31) into the areal cadence law $N^2 = 1 - C_A$ gives

$$N^2 = 1 - C_\rho + \frac{1}{2} C_\rho^2 + O(C_\rho^3). \quad (10.32)$$

Since the PPN definition is

$$N^2 = 1 - C_\rho + \frac{\beta_{\text{PPN}}}{2} C_\rho^2 + O(C_\rho^3), \quad (10.33)$$

the result is

$$\boxed{\beta_{\text{PPN}} = 1.} \quad (10.34)$$

The same result is obtained dynamically from the minimal constant-coefficient two-derivative completion

$$S_{\text{CNL-2D}}^{\text{min}} = P \int dT d^3x \sqrt{g} \left[\frac{1}{4N c_{\mathbb{E}}^2} \left(\mathcal{K}_{ij} \mathcal{K}^{ij} - \frac{1}{3} \mathcal{K}^2 \right) + N {}^{(3)}R[g] \right] + S_m, \quad (10.35)$$

where

$$\mathcal{K}_{ij} = \partial_T g_{ij} - 2D_{(i} B_{j)}, \quad P = 2Z = \frac{c_{\mathbb{E}}^4}{16\pi G}. \quad (10.36)$$

The explicit factor $c_{\mathbb{E}}^{-2}$ in the kinetic term is required to recover the CLG-1 normalisation. For a static source, $\mathcal{K}_{ij} = 0$. In the exterior the minimal action gives

$${}^{(3)}R[g] = 0, \quad D^2 N = 0. \quad (10.37)$$

With the isotropic representative

$$g_{ij} = \Omega^4 \delta_{ij}, \quad \Omega = 1 + u, \quad u = \frac{R_S}{4\rho} = \frac{GM}{2\rho c_{\mathbb{E}}^2}, \quad (10.38)$$

the regular asymptotically normalised solution is

$$N = \frac{1-u}{1+u}, \quad r_A = \rho \Omega^2 = \rho(1+u)^2. \quad (10.39)$$

It follows identically that

$$C_A = \frac{4u}{(1+u)^2}, \quad \boxed{N^2 = 1 - C_A = \left(\frac{1-u}{1+u} \right)^2.} \quad (10.40)$$

Thus the same solution possesses both the finite zero $N = 0$ at $u = 1$, equivalently $C_A = 1$, and

the isotropic expansion

$$N^2 = 1 - 2U + 2U^2 + O(U^3), \quad U = \frac{GM}{\rho c_{\mathcal{E}}^2}, \quad (10.41)$$

which is Eq. (10.34).

STATUS OF THE SECOND-ORDER CLOSURE

$\beta_{\text{PPN}} = 1$ is derived twice on the same branch: kinematically from the areal–isotropic radius map, and dynamically from the minimal CNL-2D candidate. It is not fitted and it is not a refutation. The remaining qualification concerns uniqueness, not the algebra. Because N is a physical cadence field, spatial relabelling alone can admit coefficient functions of N , such as $A(N)$, $C(N)$ or $D(N)a_i a^i$. CLG-1 fixes their required background coefficients but does not yet exclude every higher derivative with respect to N . Until a cadence minimality theorem excludes them and the nonlinear Dirac matrix is computed, CNL-2D is the minimal completion candidate and Eq. (10.34) is a conditional theorem of that branch.

With $\gamma_{\text{PPN}} = \beta_{\text{PPN}} = 1$, the perihelion factor is

$$\frac{2 + 2\gamma_{\text{PPN}} - \beta_{\text{PPN}}}{3} = 1, \quad (10.42)$$

giving the standard 42.98'' per century for Mercury.

XIII. What this note does not establish

1. **The temporal coefficient is imported, not derived here.** Eq. (10.7) rests on the registered identification $C = -2\Psi_{\mathcal{E}}/c_{\mathcal{E}}^2$ and the standard weak-field correspondence $g_{00} = 1 + 2\Psi/c^2$. This note derives the *spatial* coefficient and the symbol that joins them; it does not rederive the temporal one from an electromagnetic operator.
2. **Cadence universality (Lemma 3) is constitutive.** That the same N governs material clocks and the electromagnetic operator is the RM form of local position invariance. It is motivated by the Exclusion Interaction (historical PS-0 label) but is not a consequence of the CLG-1 action, which contains no lapse.
3. **The complete source-coupled Dirac matrix has not been computed.** Section XI verifies the first-order source variation, the sourced momentum constraint and the preservation of the weak gravitational block. A microscopic material action and the complete nonlinear bracket matrix are still required before the result can be promoted beyond T1.
4. **A genuine tension in the RM ontology is exposed.** CLG-1 §II states that spatial labels carry no physics, while the founding ontology states that $\Omega_{\mathcal{E}}$ is a physical body. Lemma 2, and with it the uniqueness of $\lambda = 1$, uses the first reading. If RM insists that the points of $\Omega_{\mathcal{E}}$ are individually physical, the labelling symmetry is not a gauge redundancy, λ

is no longer forced, and the degree count $N_{\text{phys}} = 2$ must also be re-examined. *RM cannot have both readings.* This is the principal conceptual debt created by the present note.

5. **The uniqueness of CNL-2D is not established.** Section XII derives $\beta_{\text{PPN}} = 1$ for the minimal constant-coefficient completion and from the exact areal cadence law. Spatial relabelling by itself does not yet exclude additional functions of the physical field N whose weak expansion begins at cubic order. Their exclusion must follow from an RM cadence principle, not from the desired PPN value.
6. **The numerical closures do not exhaust the PPN framework.** The values $\gamma = \beta = 1$ and $\alpha_1 = \alpha_2 = 0$ are the four parameters treated here. The remaining non-conservative PPN coefficients are not silently set to zero; their absence must follow from the complete universal matter action.
7. **The extrapolation of Eq. (10.13) to $C \rightarrow 1$ is not licensed.** $n^2 = (1 + C)/(1 - C)$ is exact only in the representative used and only if $\psi = +C$ persists, which the quadratic CLG-1 does not establish beyond linear order. Argument 1 of Section VIII is therefore a structural consistency argument, not a strong-field derivation, and it inherits a residual circularity: DSB-1's optical threshold at $C = 1$ is itself an imported Schwarzschild structure.
8. **No observational distinctness is claimed in the four coefficients computed here.** Their values coincide with the standard local values. RM's distinguishing content, if any, lies in the nonlinear saturation sector, the physical status of the labels, the homogeneous mode and the cosmological boundary-production branch.

XIV. Rejection conditions

This note fails, and the local RM branch with it, if any of the following occurs:

1. Eq. (10.9) or Eq. (10.13) fails under direct recalculation.
2. The Dirac analysis with electromagnetic sources shows that a coupling with $\lambda \neq 1$ is admissible without loss of the degree count; γ_{PPN} would then contain a free coefficient and the contract would be violated retroactively.
3. $\sigma < 0$ is shown to be excluded by an independent argument elsewhere in RM. In that case $\gamma_{\text{PPN}} = -1$, the solar deflection vanishes identically, and the local branch is refuted outright rather than recalibrated.
4. The nonlinear completion fails to preserve Eq. (10.5), removing the basis of Lemma 2.
5. A microscopic matter action compatible with the same labels changes the unit source coupling, makes the sourced momentum constraint second class, or produces non-zero α_1 or α_2 . No second vector normalisation may be introduced to repair the branch.

6. The areal–isotropic map or the static equations of the accepted completion fail to reproduce Eq. (10.40), or an independently required cadence operator gives $\beta_{\text{PPN}} \neq 1$. CNL-2D is then rejected rather than supplemented with a coefficient chosen to recover Mercury.
7. The nonlinear Dirac calculation changes the intended rank, produces a tertiary inconsistency or resurrects a scalar gravitational mode.

XV. Append-only register

Reference	Statement	Status
N-Æ-02.1	Eikonal algebraicity: only algebraic dependence on the state enters the principal symbol	T1, derived
N-Æ-02.2	Labelling-gauge rigidity: the electromagnetic coupling is $-\frac{1}{2}q_{ij}\Sigma^{ij}$ with $\lambda = 1$ unique	T1, derived
N-Æ-02.3	Cadence universality: S_{EM} of Eq. (10.10)	T1, constitutive
N-Æ-02.4	$\mathfrak{P} = \omega^2/(N^2 c_{\text{E}}^2) - g^{ij}k_i k_j$, no fourth dimension	T1, derived
N-Æ-02.5	$n^2 = (1 + \psi)/(1 - C)$ in the isotropic representative	T1, derived
N-Æ-02.6	$\gamma_{\text{PPN}} = \psi/C = -\text{sgn}(\sigma)$	T1, derived
N-Æ-02.7	$\gamma_{\text{PPN}} \in \{-1, +1\}$: no continuous optical parameter exists in RM	T1, derived
N-Æ-02.8	$\sigma > 0$ gives $n \equiv 1$: optically trivial, incompatible with DSB-1 Zone I	T1, derived
N-Æ-02.9	Selector $\text{sgn}(\sigma) = -1$, hence $\gamma_{\text{PPN}} = +1$ exactly	T1, conditional on N-Æ-02.3 and N-Æ-02.8
N-Æ-02.10	Cassini: 0.91σ , parameter-free null result	T1, confrontation
N-Æ-02.11	Gauge consistency of the material source requires $g_B = g_Q$; universal minimal coupling fixes $g_B = g_Q = 1$	T1, conditional
N-Æ-02.12	The sourced momentum constraint preserves the weak first-class gauge block and the two gravitational polarisations	T1, conditional on the material balance
N-Æ-02.13	The vector equation gives $b_i = \frac{7}{2}V_i + \frac{1}{2}W_i$	T1, derived on the minimal source branch
N-Æ-02.14	$\alpha_1 = \alpha_2 = 0$ without a vector fit	T1, conditional theorem
N-Æ-02.15	The areal and isotropic compactnesses satisfy $C_A = C_\rho - \frac{1}{2}C_\rho^2 + O(C_\rho^3)$	T1, derived from $\gamma = 1$
N-Æ-02.16	$N^2 = 1 - C_A$ gives $\beta_{\text{PPN}} = 1$ in isotropic PPN variables	T1, conditional theorem
N-Æ-02.17	Minimal CNL-2D independently gives $N = (1 - u)/(1 + u)$, $r_A = \rho(1 + u)^2$ and the same $\beta = 1$	T1, conditional on the nonlinear Dirac closure
A-Æ-02	Audit: the Section XII exclusion was conditional on an undetermined $\sigma > 0$	registered
D-Æ-02	Debt: labels cannot be both gauge and physical; reading to be arbitrated	open, conceptual
D-Æ-03	Protect the origin of CNL-2D: derive cadence minimality or classify and exclude all additional admissible functions of N	open, priority
D-Æ-04	Compute the complete nonlinear Dirac matrix with the microscopic matter action	open, priority
N-Æ-05	Superseding v4.1.0 cadence-minimality closure: $A = P/(4Nc_{\text{E}}^2)$, $B = P/(12Nc_{\text{E}}^2)$, $C = PN$, $D = V = 0$ under PCP	D-Æ-03 closed conditionally
N-Æ-06	Superseding v4.2.0 Dirac audit: universal matter interface plus minimal constrained S_A gives six first-class and ten second-class constraints on the regular $k \neq 0$ branch	D-Æ-04 narrowed; grav. chain survives

Continued on the next page

Reference	Statement	Status
D-Æ-04A	Nonlinear gravitational Dirac chain, including the minimal constrained antisymmetric sector	closed conditionally
D-Æ-04M	Explicit microscopic validation for sectors with metric/triad velocities, notably spinorial matter	open, narrowed
F-Æ-01	Spatial eikonal operator and γ_{PPN}	closed at first order , conditional
F-Æ-02	$\beta_{\text{PPN}} = 1$ and $\alpha_1 = \alpha_2 = 0$ on the branch $\sigma < 0$	closed conditionally ; D-Æ-03 is closed conditionally and residual debt D-Æ-04M remains

NON-REPAIR CLAUSE

Register entry N-Æ-01.2 and the Section XII exclusion are not rewritten. Both were correct under the restriction they carried. They are superseded, on the branch selected here, by N-Æ-02.9. If $\sigma < 0$ is later excluded, the correct conclusion is the refutation of the local RM optical branch, not the reintroduction of a free coefficient. Likewise, if the universal material completion or the cadence-minimality derivation fails, $\alpha_1 = \alpha_2 = 0$ or $\beta = 1$ must be withdrawn rather than restored by an adjustable coefficient. No optical, vector or second-order PPN parameter has been fitted, and none may be added afterwards.

One matching. One selected sign. No PPN adjustment.

$\gamma_{\text{PPN}} = 1, \quad \beta_{\text{PPN}} = 1, \quad \alpha_1 = \alpha_2 = 0$ — **conditionally derived.**

The Minimal Two-Derivative Completion — N-Æ-04

STATUS OF THIS NOTE

This note extracts the valid structural content of the proposed two-derivative completion and places it at the status supported by the calculation. It does *not* establish a unique nonlinear action. It identifies a minimal constant-coefficient candidate whose quadratic expansion recovers CLG-1, shows why the trace-free kinetic term produces a primary conformal constraint, explains the special status of the homogeneous mode, and recovers the static branch yielding $\beta_{\text{PPN}} = 1$. The two-mode count is conditional on the termination and rank of the complete nonlinear Dirac chain. Additional admissible functions of the physical cadence field N remain the constitutive debt D-Æ-03.

I. What two-derivative covariance permits

RM uses the open universal time line T and the three-dimensional physical domain $\Omega_{\text{Æ}}(T)$. No four-dimensional physical manifold is introduced. On each spatial domain define

$$g_{ij} = \delta_{ij} + q_{ij}, \quad \mathcal{K}_{ij} = \partial_T g_{ij} - 2D_{(i}B_{j)}, \quad a_i = \partial_i \ln N. \quad (10.43)$$

Spatial relabelling covariance alone permits the general two-derivative family

$$S_{2D} = \int dT d^3x \sqrt{g} \left[A(N) \mathcal{K}_{ij} \mathcal{K}^{ij} - B(N) \mathcal{K}^2 + C(N) {}^{(3)}R + D(N) a_i a^i - V(N) \right] + S_m. \quad (10.44)$$

Because N is a physical cadence field rather than a removable time coordinate, the functions in Eq. (10.44) are not forbidden by the adopted spatial gauge. CLG-1 fixes their required values and derivatives at the homogeneous background only to the order already tested. For example, the absence of $|\nabla \Lambda|^2$ in CLG-1 requires $D(1) = 0$, but it does not by itself prove $D(N) \equiv 0$: a term $D(N) = d_1(N - 1) + \dots$ begins only at cubic order. The same observation applies to higher Taylor coefficients of A , B , C and V .

More precisely, the quadratic matching fixes

$$A(1) = \frac{P}{4c_{\text{Æ}}^2}, \quad B(1) = \frac{P}{12c_{\text{Æ}}^2}, \quad C(1) = C'(1) = P, \quad D(1) = 0, \quad (10.45)$$

together with the flat-background conditions on V . It does not fix the higher data $A'(1)$, $B'(1)$, $C''(1)$, $D'(1)$, $\dots$. These data do not all affect the same observable. In a static source problem $\mathcal{K}_{ij} = 0$, so the unfixed derivatives of A and B cannot move β_{PPN} ; they first enter moving-source or dynamical sectors. By contrast, higher cadence dependence in C , D or V can alter the static nonlinear equations and must be excluded before the origin of $\beta = 1$ is called protected.

AUDIT OF THE UNIQUENESS CLAIM

The five constant-coefficient operators $\mathcal{K}_{ij}\mathcal{K}^{ij}$, $\mathcal{K}^2$, ${}^{(3)}R$, $a_i a^i$ and 1 form a useful inventory, but they do not prove uniqueness when their coefficients may be functions of the physical scalar N . Consequently the completion below is the *minimal constant-coefficient branch*. Its PPN values are consequences of that branch; the exclusion of all competing cadence functions requires a separate cadence-minimality theorem.

II. The minimal candidate CNL-2D

The smallest candidate reproducing the frozen quadratic normalisations is

CNL-2D — MINIMAL CONSTANT-COEFFICIENT BRANCH

$$S_{\text{CNL-2D}}^{\min} = P \int dT d^3x \sqrt{g} \left[\frac{1}{4N c_{\mathcal{E}}^2} \left(\mathcal{K}_{ij} \mathcal{K}^{ij} - \frac{1}{3} \mathcal{K}^2 \right) + N {}^{(3)}R[g] \right] + S_m[N, B_i, g_{ij}; \text{matter}]$$

with

$$P = 2Z = \frac{c_{\mathcal{E}}^4}{16\pi G}, \quad \sigma = -\frac{16\pi G}{c_{\mathcal{E}}^2}.$$

The factor $c_{\mathcal{E}}^{-2}$ in the kinetic term is essential: without it the quadratic coefficient does not equal $Z/(2c_{\mathcal{E}}^2)$ and the tensor front is not normalised to $c_{\mathcal{E}}$.

Let $N = 1 + \nu$. To second order,

$$\frac{P}{4N c_{\mathcal{E}}^2} \mathcal{K}_{ij}^{\text{TF}} \mathcal{K}_{\text{TF}}^{ij} \longrightarrow \frac{Z}{2c_{\mathcal{E}}^2} \mathcal{K}_{ij}^{\text{TF}} \mathcal{K}_{\text{TF}}^{ij}, \quad (10.46)$$

$$P\nu [\sqrt{g} {}^{(3)}R]_{\text{lin}} \longrightarrow -P\nu \mathcal{G}[q], \quad (10.47)$$

$$P[\sqrt{g} {}^{(3)}R]_{\text{quad}} \longrightarrow -\frac{Z}{2} \mathcal{V}_{\text{FP}}^{(3)}. \quad (10.48)$$

With $\Lambda = -P\nu$, the matter term must reproduce $-\Lambda\sigma\rho_m$. This is a condition on the complete universal matter action, not an independently completed result. Subject to it, the minimal candidate has CLG-1 as its quadratic truncation.

III. The conformal primary constraint

For a kinetic term $\mathcal{K}_{ij}\mathcal{K}^{ij} - \lambda\mathcal{K}^2$, the momentum has the form

$$\pi^{ij} = \frac{P\sqrt{g}}{2N c_{\mathcal{E}}^2} \left(\mathcal{K}^{ij} - \lambda g^{ij} \mathcal{K} \right), \quad \pi \equiv g_{ij} \pi^{ij} \propto (1 - 3\lambda) \mathcal{K}. \quad (10.49)$$

CLG-1 already selects the trace-free value $\lambda = 1/3$. Hence the Legendre map is degenerate precisely in the conformal direction and

$$\boxed{\pi = g_{ij}\pi^{ij} \approx 0} \quad (10.50)$$

is a primary constraint. The trace does not acquire an independent kinetic momentum; it is constrained rather than propagated.

Together with $p_N \approx 0$ and $p_{B_i} \approx 0$, consistency gives the schematic chains

$$p_{B_i} \longrightarrow \mathcal{H}_i, \quad (10.51)$$

$$p_N \longrightarrow \mathcal{H}_N, \quad (10.52)$$

$$\pi \longrightarrow \mathcal{C}_\pi. \quad (10.53)$$

The spatial pairs $(p_{B_i}, \mathcal{H}_i)$ generate the adopted relabelling symmetry. On the non-homogeneous branch the remaining four constraints are expected to form an elliptic second-class block. The linearised bracket contains D^2 , so its Fourier symbol contains $-k^2$ and is invertible only for $k \neq 0$. If the nonlinear chain terminates with this rank, the gravitational phase space count is

$$N_{\text{phys}} = \frac{1}{2}(20 - 2 \times 6 - 4) = 2. \quad (10.54)$$

STATUS OF THE DIRAC COUNT

Equation (10.54) is a conditional count, not a certified nonlinear theorem. The complete bracket matrix has not been evaluated on a general background, and preservation of $\mathcal{C}_\pi$ has not been shown explicitly to terminate without a tertiary constraint. A changed rank or an additional propagating scalar refutes this minimal branch rather than authorising a new fitted coefficient.

IV. Why the homogeneous mode is different

The same elliptic structure explains the standing restriction $k \neq 0$. At $k = 0$ the constraint block loses rank and the inverse Laplacian used by the local scalar closure does not exist. In the linearised secondary chain the condition takes the form

$$\nabla^2 \mathcal{K} = 0. \quad (10.55)$$

With local decaying boundary conditions it gives $\mathcal{K} = 0$. On a homogeneous cosmological branch it instead permits

$$\boxed{\mathcal{K} = 3H(T)}, \quad (10.56)$$

where $H(T)$ is spatially constant and not fixed by the local equations. This does not derive cosmological expansion. It identifies exactly where the local closure stops and where RM's boundary-production law must enter. The local sector and the cosmological mode are therefore separated by the rank of the constraint operator, not by an imposed projector.

V. The static branch and β_{PPN}

For a static source at rest in the selected $\mathcal{A}$ frame, $\mathcal{K}_{ij} = 0$. The trace-free kinetic coefficient is therefore absent from the static field equations. In the exterior vacuum the minimal branch reduces to

$${}^{(3)}R[g] = 0, \quad D^2 N = 0. \quad (10.57)$$

In isotropic radius ρ , use

$$g_{ij} = \Omega^4 \delta_{ij}, \quad \Omega = 1 + u, \quad u = \frac{R_S}{4\rho} = \frac{GM}{2\rho c_{\mathcal{A}}^2}. \quad (10.58)$$

The asymptotically normalised solution is

$$N = \frac{1-u}{1+u}, \quad r_A = \rho(1+u)^2, \quad qquad C_A = \frac{4u}{(1+u)^2}, \quad (10.59)$$

and therefore the exact identity $N^2 = 1 - C_A$ holds. Writing $U = GM/(\rho c_{\mathcal{A}}^2) = 2u$ gives

$$N^2 = 1 - 2U + 2U^2 + O(U^3), \quad qquad g_{ij} = \left(1 + 2U + \frac{3}{2}U^2 + O(U^3)\right) \delta_{ij}. \quad (10.60)$$

The same solution also gives the exact optical index of the minimal branch. Since the principal symbol contains $g^{ij} = \Omega^{-4} \delta^{ij}$,

$$n_{\min} = \frac{\Omega^2}{N} = \frac{(1+u)^3}{1-u} \quad (10.61)$$

and therefore

$$n_{\min} = 1 + C_\rho + O(C_\rho^2), \quad n_{\min} \longrightarrow \infty \quad \text{as} \quad u \rightarrow 1, r_A \rightarrow R_S, C_A \rightarrow 1. \quad (10.62)$$

Thus the weak optical result and the strict zero of the cadence belong to one closed-form solution. This is exact *within the minimal CNL-2D branch*; it is not an unconditional theorem of every action allowed by Eq. (10.44). The radius relation itself is the standard isotropic–areal relation of the static spherical vacuum solution; its value here is to remove an internal RM ambiguity, not to claim a new geometrical discovery.

Against the standard isotropic PPN expansion, this yields

STATIC PPN RESULT OF THE MINIMAL BRANCH

$$\gamma_{\text{PPN}} = 1, \quad \beta_{\text{PPN}} = 1.$$

This is the dynamical counterpart of the double-radius identity derived in the consolidated N- $\mathcal{A}$ -02. It is not a fit and it is not an observational distinction from general relativity. It is conditional on the minimal action, the selected sign $\sigma < 0$, the universal matter coupling and the unresolved nonlinear Dirac closure.

VI. What is recovered from N-Æ-04, and what is not

Recovered result	Correct status in v4.0.0
$\lambda = 1/3$ makes the trace momentum primary	Derived from the frozen trace-free kinetic term
The elliptic second-class block is singular at $k = 0$	Derived at the linearised-symbol level
$\mathcal{K} = 3H(T)$ is allowed homogeneously	Conditional local/cosmological separation; $H(T)$ not derived
The minimal static branch gives $\beta_{\text{PPN}} = 1$	Conditional theorem; independently supported by the areal-isotropic map
Exact optical index $n_{\text{min}} = (1 + u)^3/(1 - u)$	Exact within the minimal static branch; diverges at $N = 0$, $C_A = 1$
Two local tensor modes	Conditional until the full nonlinear bracket matrix and matter action are computed
Uniqueness of CNL-2D	Not established ; replaced by cadence-minimality debt D-Æ-03
$D(N) \equiv 0$ from the quadratic absence of $ \nabla\Lambda ^2$	Not established ; only the relevant background coefficient is fixed
All PPN coefficients equal their GR values	Not claimed ; only $\gamma, \beta, \alpha_1, \alpha_2$ are treated

VII. Rejection conditions and register

The minimal two-derivative branch is rejected, rather than repaired, if any of the following occurs:

1. the complete nonlinear bracket calculation changes the rank, produces a tertiary inconsistency or releases a scalar mode;
2. the microscopic matter action fails to reproduce the source terms and the universal momentum balance used in N-Æ-02;
3. an independently required cadence function changes the static equations and gives $\beta_{\text{PPN}} \neq 1$;
4. the boundary-production sector cannot supply a cosmological $H(T)$ compatible with Eq. (10.56);
5. the branch requires a rescaling of P , Z , σ or $c_{\text{Æ}}$ introduced only to recover a measured PPN value.

Reference	Statement	Status
N-Æ-04.1	CNL-2D is a minimal constant-coefficient two-derivative candidate	T0, candidate
N-Æ-04.2	$\lambda = 1/3$ gives $\pi \approx 0$ as a primary conformal constraint	T1, derived
N-Æ-04.3	The local elliptic block requires $k \neq 0$	T1, symbol derived
N-Æ-04.4	The linearised chain permits $\mathcal{K} = 3H(T)$ at $k = 0$	T1, conditional
N-Æ-04.5	The minimal static branch gives $\gamma = \beta = 1$	T1, conditional
N-Æ-04.6	The nonlinear branch carries two local modes	T1, conditional on D-Æ-04
N-Æ-04.7	$n_{\min} = (1+u)^3/(1-u)$ and $n_{\min} \rightarrow \infty$ at $N = 0$, $C_A = 1$	T1, exact within minimal static branch
D-Æ-03	Derive cadence minimality or classify the admissible functions of N	open, priority
D-Æ-04	Compute the complete nonlinear Dirac matrix with the microscopic matter action	open, priority
F-Æ-03	Reduced homogeneous $H(T)$ is supplied by the later HBR-1 bulk closure; boundary production and its coupling to the produced domain remain separate	closed at reduced bulk level / boundary open

NON-REPAIR CLAUSE

No value of γ_{PPN} , β_{PPN} , α_1 or α_2 was used to select a coefficient of Eq. (10.44). The minimal branch inherits only the normalisations already frozen in CLG-1. If cadence minimality or the nonlinear Dirac closure fails, the corresponding PPN result is withdrawn rather than restored by adding a parameter.

The Pure Cadence Principle — N-Æ-05

STATUS OF THIS NOTE

This note addresses the constitutive debt D-Æ-03 left open in Pillar II v4.0.0. It does not claim that spatial relabelling covariance alone selects a unique nonlinear action: Pillar II already proves that it does not. Instead, it extends the cadence-universality statement already used in N-Æ-02 and isolates one additional constitutive principle: the cadence field N is a rate converter, not an autonomous local energy reservoir. Under that principle, the full two-derivative dependence on N is fixed without reference to any measured PPN value. The result is therefore a *conditional constitutive theorem*. D-Æ-04 — the complete nonlinear Dirac matrix with microscopic matter — remains fully open and is the next certification step.

10.2 Problem inherited from Pillar II v4

Pillar II uses the open universal time line T and the three-dimensional physical domain $\Omega_{\mathbb{E}}(T)$. The nonlinear two-derivative variables are

$$g_{ij} = \delta_{ij} + q_{ij}, \quad \mathcal{K}_{ij} = \partial_T g_{ij} - 2D_{(i} B_{j)}, \quad a_i = \partial_i \ln N. \quad (10.63)$$

Spatial relabelling covariance by itself permits

$$S_{2D} = \int dT d^3x \sqrt{g} \left[A(N) \mathcal{K}_{ij} \mathcal{K}^{ij} - B(N) \mathcal{K}^2 + C(N) {}^{(3)}R + D(N) a_i a^i - V(N) \right] + S_m. \quad (10.64)$$

The quadratic closure fixes only the background jet

$$A(1) = \frac{P}{4c_{\mathbb{E}}^2}, \quad B(1) = \frac{P}{12c_{\mathbb{E}}^2}, \quad C(1) = C'(1) = P, \quad D(1) = 0, \quad (10.65)$$

with the corresponding flat-background conditions on V . It does not determine $A'(1)$, $B'(1)$, $C''(1)$, $D'(1)$, and the higher Taylor data. This is precisely D-Æ-03.

NO FALSE UNIQUENESS

Equation (10.64) is the correct starting point. Nothing in spatial covariance alone forbids $D(N) = d_1(N - 1) + \dots$, nor does the quadratic absence of $|\nabla \Lambda|^2$ prove $D(N) \equiv 0$. N-Æ-05 must therefore add physical content or leave D-Æ-03 open. It may not infer uniqueness from the desired value $\beta_{\text{PPN}} = 1$.

10.3 Existing input: cadence universality from N-Æ-02

N-Æ-02 already contains the needed conversion rule. The local electromagnetic law is written in local proper units and then expressed on the universal line T through

$$d\tau_{\text{cad}} = N dT. \quad (10.66)$$

Here τ_{cad} is renamed the *proper-cadence parameter*. It is not a second physical time coordinate. The sole physical time of RM remains T ; $d\tau_{\text{cad}}$ counts the local rate of a physical process relative to dT .

The same statement gives the electromagnetic action

$$S_{\text{EM}} = \frac{1}{2} \int dT d^3x \sqrt{g} \left[\frac{g^{ij} E_i E_j}{N c_{\text{Æ}}^2} - N g_{ij} B^i B^j \right]. \quad (10.67)$$

Thus one temporal derivative costs one factor N^{-1} when local proper-cadence units are rewritten in T , while a derivative-free spatial energy density receives the measure factor N .

N-Æ-05.1 — UNIVERSAL CADENCE TRANSLATION

Every local non-dissipative sector that is governed by the same RM cadence field is first written in local proper-cadence units, then translated to the universal time line through $d\tau_{\text{cad}} = N dT$. For a convected local quantity the corresponding rate operator is

$$\mathcal{D}_{\text{cad}} \equiv \frac{1}{N} (\partial_T - \mathcal{L}_B).$$

This is an extension of the constitutive cadence-universality statement already used for light; it introduces no new numerical coefficient.

10.4 The new constitutive statement: Pure Cadence

The existing four-state ontology of Pillar II is

$$\{m, \text{EM}, \text{Æ}, E\}. \quad (10.68)$$

The field N is therefore not introduced as a fifth energetic state. In N-Æ-05 we sharpen this into a falsifiable constitutive statement.

PURE CADENCE PRINCIPLE — PCP

The field N measures the local cadence of physical processes relative to the universal time T . At the local two-derivative level it has no autonomous stored-energy density and no autonomous spatial-gradient energy. Its admissible appearance in a local non-dissipative action is exhausted by: (i) the conversion $d\tau_{\text{cad}} = N dT$, and (ii) the replacement of local proper-cadence rates by $N^{-1}(\partial_T - \mathcal{L}_B)$. Any independently required term $V(N)$ or $D(N)a_i a^i$ falsifies PCP rather than being tuned away.

This principle is stronger than spatial relabelling covariance. It is not derived from CLG-1, whose quadratic action contains no lapse. Its role is exactly to decide the constitutive ambiguity that D-Æ-03 was created to expose.

10.5 Conditional theorem: the cadence dependence is fixed

Define the proper-cadence relational rate

$$\hat{\mathcal{K}}_{ij} \equiv \frac{1}{N} \mathcal{K}_{ij}, \quad \hat{\mathcal{K}}_{ij}^{\text{TF}} = \hat{\mathcal{K}}_{ij} - \frac{1}{3} g_{ij} \hat{\mathcal{K}}. \quad (10.69)$$

The quadratic relational energy already frozen by CLG-1 selects the trace-free combination and the coefficient $P/(4c_{\text{Æ}}^2)$. In local proper-cadence units the minimal two-derivative density is therefore

$$\mathcal{L}_{\text{Æ}}^{\text{cad}} = P\sqrt{g} \left[\frac{1}{4c_{\text{Æ}}^2} \hat{\mathcal{K}}_{ij}^{\text{TF}} \hat{\mathcal{K}}_{\text{TF}}^{ij} + {}^{(3)}R[g] \right]. \quad (10.70)$$

Using Eq. (10.66), the action on the universal time line becomes

$$\begin{aligned} S_{\text{Æ}}^{(2)} &= P \int dT d^3x N \sqrt{g} \left[\frac{1}{4c_{\text{Æ}}^2} \frac{\mathcal{K}_{ij}^{\text{TF}} \mathcal{K}_{\text{TF}}^{ij}}{N^2} + {}^{(3)}R[g] \right] \\ &= P \int dT d^3x \sqrt{g} \left[\frac{1}{4Nc_{\text{Æ}}^2} \left(\mathcal{K}_{ij} \mathcal{K}^{ij} - \frac{1}{3} \mathcal{K}^2 \right) + N {}^{(3)}R[g] \right]. \end{aligned} \quad (10.71)$$

Under PCP no independent $a_i a^i$ or local potential term is admitted. Comparison with Eq. (10.64) yields the full functions, not merely their background values:

N-Æ-05 — CONDITIONAL CADENCE-MINIMALITY THEOREM

$$\begin{aligned} A(N) &= \frac{P}{4Nc_{\text{Æ}}^2}, & B(N) &= \frac{P}{12Nc_{\text{Æ}}^2}, & C(N) &= PN, \\ D(N) &= 0, & V(N) &= 0. \end{aligned}$$

Consequently Eq. (10.64) collapses to the already registered minimal branch $S_{\text{CNL-2D}}^{\text{min}}$. No PPN observable is used in this selection.

10.6 The new predictions beyond the quadratic jet

The result freezes the entire Taylor tower that Pillar II v4 left open. With $N = 1 + \nu$,

$$\frac{1}{N} = 1 - \nu + \nu^2 - \nu^3 + \dots, \quad (10.72)$$

so that

$$A^{(n)}(1) = (-1)^n n! \frac{P}{4c_{\mathcal{E}}^2}, \quad (10.73)$$

$$B^{(n)}(1) = (-1)^n n! \frac{P}{12c_{\mathcal{E}}^2}, \quad (10.74)$$

$$C(1) = P, \quad C'(1) = P, \quad C^{(n)}(1) = 0 \quad (n \geq 2), \quad (10.75)$$

$$D^{(n)}(1) = 0, \quad V^{(n)}(1) = 0 \quad (n \geq 0). \quad (10.76)$$

In particular, the first previously unfixed coefficients become

$$\boxed{A'(1) = -\frac{P}{4c_{\mathcal{E}}^2}, \quad B'(1) = -\frac{P}{12c_{\mathcal{E}}^2}, \quad C''(1) = 0, \quad D'(1) = 0.} \quad (10.77)$$

These are not fitted numbers. They are the first nonlinear consequences of PCP and provide concrete entries for the future Dirac and moving-source audits.

10.7 Why D(N) and V(N) are the decisive terms

The functions A , B and C can be traced to an already existing local density under the universal cadence translation. By contrast, a term

$$\sqrt{g} D(N) a_i a^i = \sqrt{g} D(N) g^{ij} \partial_i \ln N \partial_j \ln N \quad (10.78)$$

assigns an independent local gradient energy to the cadence field itself. Likewise $\sqrt{g} V(N)$ assigns a local stored-energy density to the value of the cadence even when the relational rate and spatial curvature vanish.

PCP excludes both at the local two-derivative level because N is the rate map of already physical processes, not a new energetic component. This exclusion is intentionally severe: if either term is later required by the microscopic matter action, nonlinear constraint closure or observation, the correct conclusion is that PCP is false and D- $\mathcal{E}$ -03 reopens.

DO NOT OVERSTATE THIS RESULT

The four-state ontology alone does not mathematically prohibit an N -dependent contribution from being interpreted as part of State $\mathcal{E}$. Therefore $D = V = 0$ is not a theorem of ontology plus spatial covariance. It is the testable content of PCP. The achievement of N- $\mathcal{E}$ -05 is to reduce the former infinite functional ambiguity to one explicit constitutive yes/no statement, rather than to hide that ambiguity.

10.8 Compatibility with the existing Pillar II closures

The derived action is exactly the minimal CNL-2D branch already used in N-Æ-04. It therefore preserves, at the level already calculated,

- the trace-free kinetic ratio $B = A/3$;
- the primary conformal constraint $\pi = g_{ij}\pi^{ij} \approx 0$;
- the CLG-1 quadratic truncation and the matching $P = 2Z$;
- the static equations ${}^{(3)}R = 0$ and $D^2N = 0$;
- the conditional static result $\gamma_{\text{PPN}} = \beta_{\text{PPN}} = 1$ on the selected branch.

No part of this list proves the complete nonlinear degree count. The full bracket matrix remains D-Æ-04.

10.9 Scope boundary: local cadence versus global cosmology

PCP is a statement about the *local* two-derivative action. It does not determine the homogeneous kernel

$$\mathcal{K} = 3H(T). \quad (10.79)$$

The original N-Æ-05 note assigned $H(T)$ to an independent global sector because the local $k \neq 0$ action does not determine the homogeneous kernel. The later v4.7 reception now supplies the reduced bulk rate through HBR-1 under its declared ledger. This does not turn boundary production into a local potential $V(N)$: H and g_∂ remain distinct global operations, and neither is an autonomous two-derivative cadence energy.

PCP also does not exclude higher-derivative relational invariants such as the Born–Infeld curvature sector CNL-BI1. It states only that, at two derivatives, N itself carries no autonomous local energy operator.

10.10 Falsification and non-repair clauses

N-Æ-05 fails if any of the following occurs:

1. a complete microscopic matter action requires a cadence factor different from the universal map $d\tau_{\text{cad}} = N dT$;
2. a consistent nonlinear Dirac closure requires a nonzero $D(N)a_i a^i$ or $V(N)$ term;
3. moving-source calculations require higher functions of $A(N)$ or $B(N)$ different from the $1/N$ tower in Eq. (10.76);

4. an independently derived local constitutive law for State $\mathcal{A}$ supplies an autonomous cadence energy that cannot be rewritten in relational curvature or matter variables;
5. any coefficient in Eq. (10.71) must be changed after comparison with a PPN observable.

In cases 1–4, PCP is withdrawn and D- $\mathcal{A}$ -03 reopens. In case 5 the branch is rejected rather than refitted.

NON-REPAIR CLAUSE

The purpose of N- $\mathcal{A}$ -05 is origin protection. The values $\gamma = 1$, $\beta = 1$ and $\alpha_1 = \alpha_2 = 0$ are not inputs to the cadence-minimality derivation. If the next nonlinear audit contradicts Eq. (10.71), those weak-field closures lose their claimed constitutive protection; no additional function of N may be introduced solely to restore them.

10.11 Register

Reference	Statement	Status
N- $\mathcal{A}$ -05.1	$d\tau_{\text{cad}} = N dT$ universally translates local proper-cadence laws to T	P, inherited from N- $\mathcal{A}$ -02
N- $\mathcal{A}$ -05.2	Pure Cadence Principle: N has no autonomous local potential or gradient energy at two derivatives	P, new constitutive principle
N- $\mathcal{A}$ -05.3	$A = P/(4Nc_{\mathcal{A}}^2)$, $B = P/(12Nc_{\mathcal{A}}^2)$ and $C = PN$	T1, conditional on N- $\mathcal{A}$ -05.1
N- $\mathcal{A}$ -05.4	$D(N) = V(N) = 0$	T1, conditional on PCP
N- $\mathcal{A}$ -05.5	The full Taylor tower of the cadence functions is fixed by Eq. (10.76)	T1, conditional
D- $\mathcal{A}$ -03	Cadence minimality	closed conditionally by PCP
D- $\mathcal{A}$ -04	Complete nonlinear Dirac matrix with microscopic matter	open, next priority
F- $\mathcal{A}$ -03	Reduced homogeneous mode closed later by HBR-1; boundary production and its coupling remain open	superseded status in v4.7.0

10.12 Conclusion

Pillar II v4 correctly refused to call CNL-2D unique because spatial covariance leaves an infinite functional family in N . N- $\mathcal{A}$ -05 does not reverse that audit. It answers it with one explicit physical choice.

If the cadence field is only the local rate map of physical processes relative to the universal time T , and if the same universal cadence translation already used for light applies to the relational

sector, the factors $1/N$ and N in CNL-2D are no longer arbitrary. If, in addition, cadence carries no autonomous local stored energy, the remaining $D(N)$ and $V(N)$ operators are excluded. The infinite functional ambiguity collapses to the frozen minimal action.

VERDICT

$$\boxed{\text{Universal cadence translation} + \text{Pure Cadence} \implies S_{\text{CNL-2D}}^{\min}}$$

D-Æ-03 is therefore closed *conditionally*, not absolutely. The next non-negotiable test is D-Æ-04: compute the complete nonlinear Dirac chain with the universal microscopic matter interface and verify that the branch still carries exactly two local tensor modes.

The Universal Matter Interface and the Nonlinear Dirac Audit — N-Æ-06

STATUS OF THIS NOTE

This note performs the first nonlinear certification step requested by debt D-Æ-04 after N-Æ-05 froze the two-derivative cadence dependence. It derives a universal matter interface, obtains the canonical Hamiltonian of the frozen CNL-2D branch, and computes the complete primary/secondary scalar constraint chain for local non-homogeneous modes. The symmetric-cadence gravitational sector survives this audit: on the regular branch where one explicit elliptic operator has no kernel, the chain terminates with six spatial first-class constraints and four scalar second-class constraints. A minimal covariant completion of the already constrained antisymmetric CLG-1 sector adds three nondynamical second-class pairs and no new coefficient. The resulting local relational bookkeeping therefore recovers six first-class and ten second-class constraints, leaving exactly two local gravitational degrees of freedom. This is not yet a certification of every microscopic matter species or of arbitrary strong antisymmetric backgrounds; those residual items are isolated at the end rather than hidden.

10.13 Frozen input from N-Æ-05

The local two-derivative relational action is no longer varied during this audit:

FROZEN CNL-2D BRANCH

$$S_g = P \int dT d^3x \sqrt{g} \left[\frac{1}{4Nc_{\mathbb{E}}^2} \left(\mathcal{K}_{ij} \mathcal{K}^{ij} - \frac{1}{3} \mathcal{K}^2 \right) + N {}^{(3)}R[g] \right],$$

$$\mathcal{K}_{ij} = \partial_T g_{ij} - 2D_{(i} B_{j)}, \quad P = \frac{c_{\mathbb{E}}^4}{16\pi G}.$$

N-Æ-05 also freezes $D(N) = V(N) = 0$. No term is added below to improve the constraint algebra.

10.14 Universal microscopic matter interface

Let χ^A denote any local matter variables whose microscopic Lagrangian contains no time derivative of N , B_i or g_{ij} . Define the universal proper-cadence derivative

$$\mathcal{D}_T \chi^A \equiv \frac{1}{N} \left(\partial_T \chi^A - \mathcal{L}_B \chi^A \right). \quad (10.80)$$

The most general minimally relabelled local matter action compatible with the cadence map used in N-Æ-02 and N-Æ-05 can then be written

$$S_m = \int dT d^3x N \sqrt{g} \mathcal{L}_m \left(\chi^A, \mathcal{D}_T \chi^A, D_i \chi^A; g_{ij} \right). \quad (10.81)$$

No numerical gravitational coupling appears in Eq. (10.81). The same N , B_i and g_{ij} seen by the relational sector are used by matter.

The canonical matter momentum is

$$p_A = \frac{\partial(N\sqrt{g} \mathcal{L}_m)}{\partial(\partial_T \chi^A)} = \sqrt{g} \frac{\partial \mathcal{L}_m}{\partial(\mathcal{D}_T \chi^A)}. \quad (10.82)$$

The factors of N cancel. Therefore the Legendre transform is affine in N and B_i :

UNIVERSAL MATTER HAMILTONIAN

$$H_m = \int d^3x \left(N \mathcal{H}_m + B^i \mathcal{H}_i^m \right) + H_m^{\text{internal constraints}}.$$

This is the nonlinear origin of the unit first-order source coefficients. Expanding the same action about $N = 1$, $B_i = 0$, $g_{ij} = \delta_{ij} + q_{ij}$ produces one functional derivative with respect to B_i and one with respect to g_{ij} ; no independent constants g_B and g_Q exist to be adjusted. Thus the N-Æ-02 condition $g_B = g_Q = 1$ is reproduced by the interface rather than imposed after variation.

Explicit carrier check

For a real scalar carrier,

$$S_\phi = \int dT d^3x N \sqrt{g} \left[\frac{1}{2c_\phi^2} (\mathcal{D}_T \phi)^2 - \frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - U(\phi) \right], \quad (10.83)$$

one finds

$$p_\phi = \frac{\sqrt{g}}{c_\phi^2} \mathcal{D}_T \phi, \quad (10.84)$$

$$H_\phi = \int d^3x \left[N \left(\frac{c_\phi^2 p_\phi^2}{2\sqrt{g}} + \sqrt{g} \left(\frac{1}{2} D_i \phi D^i \phi + U \right) \right) + B^i p_\phi \partial_i \phi \right]. \quad (10.85)$$

The affine structure is explicit and no autonomous cadence energy is generated.

10.15 Canonical gravitational variables

Define the traceless momentum

$$\pi^{ij} = \frac{\partial \mathcal{L}_g}{\partial (\partial_T g_{ij})} = \frac{P\sqrt{g}}{2Nc_{\mathbb{E}}^2} \mathcal{K}_{\text{TF}}^{ij}. \quad (10.86)$$

Taking the trace gives the primary conformal constraint

$$\boxed{\pi \equiv g_{ij} \pi^{ij} \approx 0.} \quad (10.87)$$

Since N and B_i carry no time derivatives,

$$\boxed{p_N \approx 0, \quad p_i^B \approx 0.} \quad (10.88)$$

Solving only the traceless velocity gives

$$\mathcal{K}_{\text{TF}}^{ij} = \frac{2Nc_{\mathbb{E}}^2}{P\sqrt{g}} \pi^{ij}. \quad (10.89)$$

The trace velocity remains a Lagrange multiplier, as required by Eq. (10.87).

10.16 Exact canonical Hamiltonian

Up to a spatial boundary term, the total Hamiltonian can be written

NONLINEAR CANONICAL FORM

$$\boxed{H_T = \int d^3x \left[N\mathcal{H}_0 + B^i \mathcal{H}_i + \lambda\pi + u_N p_N + u_B^i p_i^B \right] + H_m^{\text{internal constraints}},}$$

$$\boxed{\mathcal{H}_0 = \frac{c_{\mathbb{E}}^2}{P\sqrt{g}} \pi^{ij} \pi_{ij} - P\sqrt{g} {}^{(3)}R + \mathcal{H}_m,}$$

$$\boxed{\mathcal{H}_i = -2D_j \pi^j_i + \mathcal{H}_i^m.}$$

The key structural fact is that PCP and the universal matter interface make H_T linear in N and B_i without making either field dynamical.

10.17 Secondary constraints

Preservation of p_i^B gives

$$\mathcal{H}_i \approx 0. \quad (10.90)$$

By time-dependent spatial relabelling covariance, the six constraints $(p_i^B, \mathcal{H}_i)$ form the spatial first-class block, conditional on the gauge reading of the labels already used in N-Æ-02.

Preservation of p_N gives the nonlinear scalar constraint

$$\boxed{\mathcal{H}_0 \approx 0.} \quad (10.91)$$

To preserve π , define the matter conformal response operator $\Theta_m[f]$ by the variation of the smeared matter Hamiltonian under $\delta g_{ij} = \omega g_{ij}$ with matter canonical variables held fixed:

$$\delta_\omega \int d^3x f \mathcal{H}_m \equiv \int d^3x \omega \Theta_m[f]. \quad (10.92)$$

For minimally coupled matter, $\Theta_m[f]$ does not alter the second-order gravitational principal part. Introduce

$$\mathfrak{L}[f] \equiv \frac{3}{2} f \frac{c_{\mathbb{E}}^2}{P\sqrt{g}} \pi^{ij} \pi_{ij} + \frac{P}{2} f \sqrt{g} {}^{(3)}R - 2P\sqrt{g} D^2 f - \Theta_m[f]. \quad (10.93)$$

Then the conformal preservation condition is

$$\boxed{\mathcal{C} \equiv \mathfrak{L}[N] \approx 0.} \quad (10.94)$$

In vacuum this reduces to

$$\frac{3}{2} N \frac{c_{\mathbb{E}}^2}{P\sqrt{g}} \pi^{ij} \pi_{ij} + \frac{P}{2} N \sqrt{g} {}^{(3)}R - 2P\sqrt{g} D^2 N \approx 0. \quad (10.95)$$

Using $\mathcal{H}_0 = 0$, the vacuum pair may equivalently be written

$$D^2 N - N {}^{(3)}R \approx 0. \quad (10.96)$$

For the static exterior branch, $\pi^{ij} = 0$ and ${}^{(3)}R = 0$, so Eq. (10.96) reduces exactly to $D^2 N = 0$, reproducing N-Æ-04.

10.18 The nonlinear scalar Dirac block

Use the ordered set

$$\chi_A = (p_N, \pi, \mathcal{H}_0, \mathcal{C}). \quad (10.97)$$

Two brackets vanish identically:

$$\{p_N, \pi\} = 0, \quad \{p_N, \mathcal{H}_0\} = 0, \quad (10.98)$$

because the frozen Hamiltonian density $\mathcal{H}_0$ contains no N after the Legendre transform. The two decisive brackets are generated by the same operator $\mathfrak{L}$:

$$\{\pi, \mathcal{H}_0[f]\} = \mathfrak{L}[f], \quad \{p_N, \mathcal{C}\} = -\mathfrak{L}, \quad (10.99)$$

up to the conventional sign associated with the smearing order. The remaining brackets $\{\pi, \mathcal{C}\}$ and $\{\mathcal{H}_0, \mathcal{C}\}$ may be nonzero, but they do not control the principal rank because of the antisymmetric pattern

$$\Omega_{\text{sc}} \sim \begin{pmatrix} 0 & 0 & 0 & -\mathfrak{L} \\ 0 & 0 & \mathfrak{L} & \star \\ 0 & -\mathfrak{L} & 0 & \star \\ \mathfrak{L} & -\star & -\star & 0 \end{pmatrix}. \quad (10.100)$$

The principal symbol of Eq. (10.93) is

$$\sigma_{\text{pr}}(\mathfrak{L})(k) = 2P\sqrt{g}g^{ij}k_ik_j. \quad (10.101)$$

Therefore

DIRAC PRINCIPAL DETERMINANT

$$\det \sigma_{\text{pr}}(\Omega_{\text{sc}})(k) = \left[2P\sqrt{g}g^{ij}k_ik_j \right]^4.$$

For $P > 0$, positive spatial g_{ij} and $k \neq 0$, the scalar block has rank four.

Matter changes lower-order coefficients in $\mathfrak{L}$ but not this principal symbol as long as the microscopic interface is minimal and introduces no curvature or cadence-gradient term of its own.

10.19 Termination of the Dirac chain

Once $\mathfrak{L}$ has trivial kernel on the chosen local boundary-value problem, preservation of $\mathcal{H}_0$ fixes the conformal multiplier λ and preservation of $\mathcal{C}$ fixes u_N (or the equivalent linear combination). They do not generate an independent tertiary constraint. The chain is therefore

$$p_N \rightarrow \mathcal{H}_0, \quad \pi \rightarrow \mathcal{C}, \quad p_i^B \rightarrow \mathcal{H}_i, \quad (10.102)$$

and terminates locally on the regular non-homogeneous branch.

RANK CONDITION - NOT A FIT

The surviving branch condition is not a new coefficient. It is the spectral statement $\ker \mathfrak{L} = \{0\}$ for the local $k \neq 0$ boundary-value problem. At the homogeneous flat background, $\mathfrak{L} = -2P\nabla^2$, so every Fourier mode $k \neq 0$ is automatically regular. A zero mode appearing on a nonlinear background would mark a genuine bifurcation of the constraint structure and would reopen or refute this branch; it may not be removed by changing the action.

10.20 Minimal covariant completion of the constrained antisymmetric sector

Pillar II requires the nonlinear theory to retain the CLG-1 antisymmetric sector $a_{ij} = Q_{[ij]}$ as constrained rather than propagating. N-Æ-06 can complete that requirement without introducing a new constant. The minimal relabelling-covariant and PCP-compatible continuation of the frozen quadratic term is

$$S_A^{\min} = -\frac{\mu_A}{4} \int dT d^3x N \sqrt{g} D_k a_{ij} D^k a^{ij} + S_A^{\text{source}}. \quad (10.103)$$

At $N = 1$ and $g_{ij} = \delta_{ij}$ this is exactly the spatial-gradient term already present in CLG-1. No $\partial_T a_{ij}$ term is generated, so the three independent antisymmetric components have primaries

$$p_A^{ij} \approx 0. \quad (10.104)$$

Their preservation gives three elliptic secondary equations

$$\mathcal{A}^{ij} = \frac{\mu_A}{2} D_k (N \sqrt{g} D^k a^{ij}) + \mathcal{J}_A^{ij} \approx 0, \quad (10.105)$$

where $\mathcal{J}_A^{ij}$ denotes the source variation. Around the intended local branch $a_{ij} = 0$, the principal bracket of each pair is

$$\sigma_{\text{pr}}(\{p_A, \mathcal{A}\}) = \frac{\mu_A}{2} N \sqrt{g} g^{ij} k_i k_j. \quad (10.106)$$

Thus $\mu_A \neq 0$, $N > 0$ and $k \neq 0$ give three second-class pairs and no antisymmetric propagating oscillator.

Combining them with the four scalar second-class constraints of Eq. (10.100), the principal determinant of the ten-dimensional second-class block has the scaling

$$\det \sigma_{\text{pr}}(\Omega_{\text{SC}}^{\text{full}}) \propto P^4 \mu_A^6 N^6 g^5 (k^2)^{10} \neq 0, \quad (10.107)$$

where the overall numerical prefactor depends only on the canonical normalization chosen for the three independent components of a_{ij} . The physically relevant statement is the rank: ten. The exponent $(k^2)^{10}$ is the same derivative-rank signature carried by the quadratic CLG-1 audit.

DOMAIN OF THE ANTISYMMETRIC RESULT

Equation (10.107) certifies the branch continuously connected to $a_{ij} = 0$. On a large nonlinear antisymmetric background, cross-brackets with the scalar sector can modify lower-order parts of the operator. A spectral zero there would be a genuine branch change and must not be repaired by adding an antisymmetric kinetic term.

10.21 Degree count

The complete local relational configuration space now contains

$$g_{ij} \ (6), \quad a_{ij} \ (3), \quad N \ (1), \quad B_i \ (3), \quad (10.108)$$

for thirteen configuration variables and twenty-six phase-space variables. On the regular local branch there are

- six first-class constraints: $(p_i^B, \mathcal{H}_i)$;
- four scalar second-class constraints: $(p_N, \pi, \mathcal{H}_0, \mathcal{C})$;
- six antisymmetric second-class constraints: three pairs $(p_A, \mathcal{A})$.

Hence

FULL NONLINEAR LOCAL GRAVITATIONAL COUNT

$$N_{\text{grav}} = \frac{1}{2} (26 - 2 \times 6 - 10) = 2.$$

Matter carries its own physical degrees of freedom and any internal gauge constraints of its microscopic sector; those do not constitute additional gravitational polarisations.

10.22 Relation to the CLG-1 rank-ten statement

CLG-1 also used thirteen local relational configuration variables: six symmetric components, three antisymmetric components, three shift variables and one scalar multiplier. In the nonlinear CNL-2D bookkeeping the physical cadence field N replaces the old quadratic multiplier architecture. The scalar second-class content remains four constraints, while Eq. (10.103) preserves the three nondynamical antisymmetric pairs. The full nonlinear local count therefore reproduces the same structural target: six first-class and ten second-class constraints, but now from the frozen cadence action rather than from the quadratic multiplier Λ .

10.23 What has survived, and what is still open

Item	Result	Status
Universal matter interface	$S_m = \int N \sqrt{g} \mathcal{L}_m(\chi, \mathcal{D}_T \chi, D\chi; g)$ and $H_m = N \mathcal{H}_m + B^i \mathcal{H}_i^m$	T1 conditional
Unit matter source	No independent g_B, g_Q appear; weak expansion gives the same unit interface	T1 conditional
Primary constraints	$p_N \approx 0, p_i^B \approx 0, \pi \approx 0$	T1 derived
Secondary constraints	$\mathcal{H}_0 \approx 0, \mathcal{H}_i \approx 0, \mathcal{C} = \mathfrak{L}[N] \approx 0$	T1 derived
Scalar rank	Principal determinant $[2P \sqrt{g} g^{ij} k_i k_j]^4$	T1 for $k \neq 0$
Local graviton count	Two gravitational modes on $\ker \mathfrak{L} = 0$ branch	T1 conditional
Minimal antisymmetric S_A	Three nondynamical pairs; full local principal rank becomes ten	T1 conditional
Microscopic fermions	Possible metric/triad-velocity subtleties not treated explicitly here	open
Strong-field spectral crossings	A zero of $\mathfrak{L}$ or of the antisymmetric elliptic block would change rank	open/testable

10.24 Falsification and non-repair clauses

This N-Æ-06 branch fails if any of the following occurs:

1. a microscopic matter sector cannot be written with the universal cadence derivative without introducing an independent gravitational coupling;
2. matter necessarily gives N, B_i or g_{ij} an additional gravitational velocity that destroys the primary constraints;
3. $\mathfrak{L}$ develops an unavoidable local kernel on the intended physical branch, releasing a scalar or producing an inconsistent chain;
4. the minimal covariant antisymmetric completion in Eq. (10.103) develops an unavoidable kernel or requires a time-derivative term, releasing an antisymmetric mode;
5. saving any of the above requires adding $D(N), V(N)$ or changing the frozen coefficients of N-Æ-05.

Items 1–4 reopen D-Æ-04 or reject the affected branch. Item 5 rejects PCP/CNL-2D rather than repairing it.

10.25 Register and verdict

Reference	Statement	Status
N-Æ-06.1	Universal minimal matter interface is affine in N and B_i after Legendre transform	T1 conditional
N-Æ-06.2	The interface reproduces the unit matter-current/stress coupling without g_B, g_Q fitting	T1 conditional
N-Æ-06.3	Frozen CNL-2D has primaries p_N, p_i^B, π and secondaries $\mathcal{H}_0, \mathcal{H}_i, \mathcal{C}$	T1 derived
N-Æ-06.4	The scalar block has principal determinant $[2P\sqrt{g}g^{ij}k_ik_j]^4$	T1 derived
N-Æ-06.5	Minimal covariant S_A keeps three antisymmetric pairs second class	T1 conditional
N-Æ-06.6	Full local relational count is six first-class plus ten second-class, hence $N_{\text{grav}} = 2$	T1 conditional
D-Æ-04A	Nonlinear gravitational Dirac chain, including minimal constrained antisymmetric sector	closed conditionally
D-Æ-04M	Explicit microscopic validation for sectors with metric/triad velocities (notably spinorial matter)	open, narrowed

VERDICT

frozen CNL-2D + universal matter $\implies$ two local gravitational modes

Provided $k \neq 0$, $\mu_A \neq 0$ and the two elliptic blocks have no kernel, the full local relational gravitational sector survives the nonlinear Dirac audit with the historical target of six first-class plus ten second-class constraints. The remaining debt is narrower: explicit microscopic validation of matter sectors carrying metric/triad velocities, and strong-field spectral stability.

P–CIN — Local Canonical Audit of the Deviatoric Capacity

The strong-field programme introduces a separate trace-free kinetic capacity candidate, P–CIN. It shares the square-root motif of the Exclusion response but is not derived from the static curvature energy $U(X)$. Its local deviatoric density is

$$\mathcal{L}_{\text{dev}}^{\text{P-CIN}} = \frac{2P}{\ell_\star^2} N \sqrt{g} [1 - \sqrt{1 - y}], \quad y = \frac{\ell_\star^2}{4c_\mathbb{E}^2} \left\| \frac{K_{\text{TF}}}{N} \right\|^2. \quad (10.109)$$

The admissible local branch is $0 \leq y < 1$.

At a local orthonormal point, write the five trace-free velocities as v_A . The conjugate momentum is

$$p_A = \frac{P}{2c_\mathbb{E}^2} \frac{v_A/N}{\sqrt{1 - y}}. \quad (10.110)$$

The map is globally invertible for every finite momentum:

$$\left\| \frac{v}{N} \right\| = \frac{2c_\mathbb{E}^2 \|p\|}{\sqrt{P^2 + c_\mathbb{E}^2 \ell_\star^2 \|p\|^2}}. \quad (10.111)$$

Thus finite momentum never crosses the rate boundary $2c_\mathbb{E}/\ell_\star$.

The Legendre transform gives

$$\mathcal{H}_{\text{dev}}^{\text{P-CIN}} = N h(\|p\|), \quad h(\|p\|) = \frac{2}{\ell_\star^2} \left[\sqrt{P^2 + c_\mathbb{E}^2 \ell_\star^2 \|p\|^2} - P \right]. \quad (10.112)$$

It is positive, single-branch and convex. The lapse remains linear and therefore non-kinetic. The kinetic Hessian has one longitudinal and four transverse positive eigenvalues for $y < 1$; they stiffen rather than change sign as $y \rightarrow 1^-$. No new primary constraint is generated in the isolated deviatoric sector.

P–CIN–DIRAC — LOCAL RESULT

The P–CIN radical is canonically healthy in the isolated local trace-free kinetic sector: regular inversion, positive convex Hamiltonian, positive Hessian, no kinetic ghost and non-dynamical lapse.

SCOPE OF THE RESULT

This pointwise audit does not prove the complete strong-field theory. It does not by itself establish that the full coupled constraint chain keeps the same rank and class, nor does it independently prove two physical polarisations for the full P–CIN system. The separated-point brackets, trace and curvature coupling, matter coupling and hyperbolicity of the evolution equations remain part of P–CIN–FIELD.

This result is sufficient to let the constructive strong-field programme use P-CIN as a locally healthy foundation while keeping its full field completion explicitly open.

Constitutive Length and the Stellar Q -Fold

STATUS OF THIS CHAPTER

This chapter is an interface, not a replacement for the N-EMPR notes. It imports the frozen v26–v28 static spherical equations and their reduced-unit numerical sequence into the technical authority of Pillar II. N-EMPR-09 is used only where explicitly labelled *pre-audit*. The unresolved global-radius discrepancy between the v28 collocation and the independent shooting calculation is preserved as a live contradiction.

I. Why the stellar calculation belongs here

Pillar I records the methodological ascent from matter imprint to the stellar fold. Pillar II is the technical authority for the local gravitational sector. The two therefore meet at one precise object: the nonlinear constitutive response already present in the action through the length $\ell_\star$ and the barrier function $F(X)$.

The key distinction is now explicit. Matter supplies the source and its equation of state. It does not choose the geometric response law. RM geometry already carries a constitutive length before the EOS is selected. In the static spherical reduction used by N-EMPR,

$$a \equiv \frac{f'}{r}, \quad b \equiv \frac{1-f}{r^2} - \frac{a}{2}, \quad \boxed{X_{\mathcal{R}} = \ell_\star^4(a^2 + 2b^2)}. \quad (10.113)$$

Equivalently, if $L_{\mathcal{R}}$ denotes the local curvature length defined by $\mathcal{R}_{ij}\mathcal{R}^{ij} = L_{\mathcal{R}}^{-4}$, then

$$\boxed{X_{\mathcal{R}} = \left(\frac{\ell_\star}{L_{\mathcal{R}}}\right)^4}. \quad (10.114)$$

Thus $L_{\mathcal{R}} \gg \ell_\star$ is a weak constitutive regime, whereas $L_{\mathcal{R}} \sim \ell_\star$ makes the nonlinear sector order unity. This is an action-level statement. It does not calibrate $\ell_\star$ in kilometres.

II. The Exclusion Interaction is already in CNL–BI1

The frozen constitutive function is

$$\boxed{F(X) = 1 - \sqrt{1-X} - \frac{X}{2}}, \quad 0 \leq X < 1, \quad (10.115)$$

with

$$F'(X) = \frac{1}{2\sqrt{1-X}} - \frac{1}{2}, \quad \boxed{X \rightarrow 1^- \implies F'(X) \rightarrow \infty}. \quad (10.116)$$

The name **Exclusion Interaction** is assigned to this increasingly stiff constitutive feedback. It is not the Pauli exclusion principle, not a fifth force and not a new fitted coefficient. It names the response already present in Eqs. (10.115)–(10.116): continued loading of the same geometric channel becomes progressively harder as the admissible constitutive boundary is approached.

CONSTITUTIVE READING

$$\boxed{\text{matter loads} \implies \text{geometry answers.}}$$

The EOS controls how matter responds to compression; the Exclusion Interaction controls how the RM geometric sector responds to curvature loading.

III. Hydrostatic closure and the equation that must not be divided

For the minimal matter interface, the static perfect-fluid identity is

$$\boxed{p' = -(\varepsilon + p) \frac{N'}{N}.} \quad (10.117)$$

The N-EMPR definitions are

$$Z(\chi) = \chi^4, \quad G = u_\star \ell_\star^4 F'(X_{\mathcal{R}}), \quad \boxed{Q = PZ + G(a - b)}. \quad (10.118)$$

The central lesson of v28 is algebraic rather than phenomenological. The cadence equation must be retained before division by Q :

$$\boxed{Q \frac{N'}{N} = Q \frac{f'}{2f} + \frac{r(\varepsilon + p)}{4f} + \frac{r\kappa_\chi \chi'^2}{4} - Q' - \frac{G(a + b)}{r}.} \quad (10.119)$$

For the Hamiltonian constraint $H = 0$ solved locally with respect to $a = f'/r$,

$$\boxed{\frac{\partial H}{\partial a} = -2Q.} \quad (10.120)$$

Hence $Q = 0$ is exactly where a loses quality as an algebraic coordinate. It is not, by itself, a physical termination of the solution. Dividing Eq. (10.119) by Q promoted a loss of parametrisation into an artificial singularity.

The standard non-regression test is unchanged: with $Z = 1$, $G = 0$, $\chi' = 0$ and $Q = P$, the system reduces exactly to the ordinary TOV mass and pressure equations in the action conventions of Pillar II. No RM-specific TOV coefficient is inserted.

IV. Frozen v28: two hydrostatic branches and the local fold

The reduced validation EOS is

$$p = \rho^2, \quad \varepsilon = \rho + p. \quad (10.121)$$

At the same central pressure

$$\boxed{p_c = 0.5028,} \quad (10.122)$$

v28 reports two converged static solutions:

Branch	X_c	Q_c	M	R	$2M/R$
$Q_c > 0$	0.899396	+0.051350	0.832632	2.112721	0.788208
$Q_c < 0$	0.910971	-0.059808	0.850506	2.046754	0.831077

These are reduced units with a toy EOS. They are not kilometres or solar masses.

Using X_c as continuation parameter, the local reconstruction around the fold gives

$$\boxed{X_{c,\star} = 0.9052492489, \quad \varepsilon_{c,\star} = 1.2120324954, \quad \left. \frac{d\varepsilon_c}{dX_c} \right|_{\star} = 0.} \quad (10.123)$$

The last accepted high-branch point frozen in v28 is

$$\boxed{X_c = 0.9715, \quad M = 0.972448, \quad R = 1.949276, \quad \frac{2M}{R} = 0.997753, \quad f_{\min} = 9.94 \times 10^{-3} > 0.} \quad (10.124)$$

No horizon, $X = 1$ endpoint or maximum RM mass is inferred from this point.

V. What the independent shooting pre-audit adds — and what it does not

N-EMPR-09 was produced from an independent DOP853 shooting calculation at relative tolerance 10^{-11} . On the $Q_c < 0$ branch it reports

$$\chi_c = 1.201356564122, \quad R = 2.08849775, \quad m(R) = 0.84650677, \quad M = 0.84685706. \quad (10.125)$$

The interior has a zero of Q at

$$\boxed{r_Q = 0.5148433, \quad r_Q/R = 0.2465,} \quad (10.126)$$

while the pressure reaches

$$\boxed{p_{\max} = 0.5028824 \quad \text{at} \quad r = 0.4982.} \quad (10.127)$$

For $r < r_Q$, the reported profile remains in the narrow ranges $X_{\mathcal{R}} \simeq 0.905\text{--}0.911$ and $\chi \simeq 1.2006\text{--}1.2014$. In that region $Q < 0$, i.e.

$$|G(a - b)| > PZ. \quad (10.128)$$

This motivates the descriptive phrase **exclusion-dominated core**: the nonlinear constitutive contribution dominates the radial combination entering Q . It does not mean anti-gravity or a new repulsive force.

The same pre-audit finds that the crossing $Q = 0$ is smooth. Around r_Q , the derived quantities a' , χ'' and N'/N vary continuously, and the compatibility remainder of Eq. (10.119) is reported at 10^{-14} . The pressure maximum occurs slightly *inside* the $Q = 0$ surface because N'/N is weakly negative before changing sign; the magnitude of that inversion is of order 10^{-4} .

HISTORICAL v4.3 AUDIT FRONTIER — SUPERSEDED IN v4.5.0

The 10 September pre-audit compared the DOP853 value $R = 2.08849775$ with the frozen v28 collocation value $R = 2.046754$ and correctly retained the disagreement rather than averaging it away. Version v4.5.0 preserves that event as history. A new, separately implemented two-domain collocation now gives $R = 2.0884976108$ and $M = 0.8468571424$ at the same $p_c = 0.5028$, reproducing the DOP853 branch and its smooth internal $Q = 0$ crossing. The v28 coordinates remain frozen as the dated discovery record; the branch-existence contradiction at this test point is no longer open.

VI. CA-1 — candidate certification protocol for stellar solvers

The contradiction above is useful because it localises what two solvers must report before their global numbers are compared. N-EMPR-09 proposes the following five-item interface. It is adopted here as an *audit protocol*, not as a physical law.

1. **Surface data.** Report $\chi(R)$ and $\chi'(R)$ with at least six decimals. Two implementations are provisionally considered to have the same surface matching only if $|\Delta\chi_R| < 10^{-3}$ and $|\Delta\chi'_R| < 5 \times 10^{-3}$.
2. **Admissible branch.** Verify $\min \chi \geq 1$ over the complete interior and exterior domain used by the calculation. A crossing below one is outside the N-EMPR-01 branch and is not treated as a small numerical correction.
3. **Mass ledger.** Report $m(R)$, the asymptotic mass M , and $M - m(R)$ separately. In the reference shooting doublet the exterior contribution is of order 10^{-4} ; a value larger than 10^{-3} requires explicit explanation, and M must be stable under extension of the exterior domain.
4. **Equation closures.** Report the Hamiltonian residual, the undivided cadence residual and the exact enthalpy identity. When Q crosses zero, also report the compatibility remainder of Eq. (10.119) at r_Q .

5. **Interior anatomy.** Report $p_{\max}$, its location and r_Q when it exists. If two solvers agree on these quantities but not on R or M , the discrepancy has been pushed to the surface matching or exterior tail rather than the interior equations.

The numerical thresholds above originate in one particularly sensitive doublet and may require revision when a physical EOS or different scaling is introduced. The invariant content of CA-1 is the separation of interior, surface and exterior diagnostics.

VI.A. v4.5.0 cross-method result at $p_c = 0.5028$

The post-v28 DOP853 and two-domain collocation calculations now agree on the regular $Q_c < 0$ solution:

$$R \simeq 2.0884976, \quad M \simeq 0.8468571. \quad (10.129)$$

The collocation centre is $X_c = 0.9109684074$, $Q_c = -0.0598152950$ and its maximum Hamiltonian residual is below 8×10^{-14} in both domains. The DOP853 anatomy remains $r_Q \simeq 0.5148433$ and $p_{\max} \simeq 0.5028824$ near $r \simeq 0.4982$. This is sufficient to promote branch existence and regularity at the declared pressure from open contradiction to cross-method conditional numerical result.

CA-1 STATUS AFTER v4.5.0

CA-1 remains active, but its role is narrower. It is no longer needed to decide whether a regular $Q_c < 0$ solution exists at $p_c = 0.5028$. It remains the certification interface for surface data, exterior mass convergence, admissibility, equation closure, extension along the high branch, physical EOS calculations and future independent human reproduction.

VII. Status and non-repair clauses

Object	Current statement in v4.5.0	Status
Constitutive length $\ell_\star$	Enters $X_{\mathcal{R}}$ before the EOS and sets the nonlinear curvature scale	action-level input
Exclusion Interaction	Canonical name for the stiff response $F'(X) \rightarrow \infty$ as $X \rightarrow 1^-$	constitutive interpretation
TOV-RM v26	Hydrostatic closure from the minimal matter interface; standard TOV recovered exactly when RM sectors are switched off	conditional derivation
v28 doublet	Two static reduced-unit solutions at the same $p_c = 0.5028$	frozen conditional numerical result
v28 fold	$X_{c,\star} = 0.9052492489$, $\varepsilon_{c,\star} = 1.2120324954$ from the local reconstruction	frozen numerical reconstruction
N-EMPR-09 interior anatomy	Smooth $Q = 0$ crossing and candidate exclusion-dominated core from DOP853 shooting	retained pre-audit evidence
$Q_c < 0$ global R, M	DOP853 and two-domain collocation agree at $p_c = 0.5028$: $R \simeq 2.0884976$, $M \simeq 0.8468571$	cross-method conditional result
External human reproduction	No such reproduction claimed in this edition; the present cross-check is between independently implemented numerical methods	open audit layer
Physical neutron-star prediction	No $\text{km}/M_\odot$ calibration and no physical EOS in the frozen sequence	outside current claim

NON-REPAIR RULE

v28 stays frozen; later agreement does not rewrite the dated record.

The new post-v28 agreement updates the audit status but does not alter the historical v28 coordinates. A future independent implementation may confirm or challenge the cross-method result, extend it along the high branch, or identify a boundary-condition limitation. None of those outcomes authorises a fitted coefficient or a post-hoc change of the frozen EOS.

- The cross-method branch result fails if an additional certified implementation of the undivided equations cannot reproduce a regular $Q_c < 0$ solution near the stated pressure with simultaneous closure of the constraint and matching conditions.
- The smooth-crossing interpretation fails if a solver satisfying the audit protocol exhibits a genuine discontinuity or rank loss at $Q = 0$.

- The exclusion-dominated-core description fails if a certified $Q_c < 0$ solution has no finite $Q < 0$ interior region or no associated change of root of the spherical constraint.
- No statement in this chapter establishes radial stability, an event horizon, $X = 1$, a maximum RM mass or an astrophysical neutron-star curve.

N-OPT-01.1 — Photonic Cadence as a Dynkin Cost

I. Scope

This chapter receives a post-v4.4.1 result without changing the electromagnetic eikonal derivation already present in Book II. The optical index, the spatial operator and the cadence field must not be collapsed into one phenomenological coefficient. The statement is instead conditional on a local diffusive regime in which matter microphysics supplies a spatial Fick operator.

Let

$$\boxed{\mathcal{L}_{\text{loc}} f = \nabla_i \left(D_{\text{loc}} \nabla^i f \right)} \quad (10.130)$$

be the generator of the local material random walk with respect to the proper-cadence parameter τ_{cad} . Book II already uses

$$\boxed{d\tau_{\text{cad}} = N dT, \quad N > 0,} \quad (10.131)$$

where T is the single physical time line of RM. The parameter τ_{cad} counts the local rate of a process; it is not a second physical time and is not photon proper time.

II. Dynkin conversion to universal time

If the local generator is Eq. (10.130), the same stochastic path parameterised by T has generator

$$\boxed{\mathcal{L}_T = N \mathcal{L}_{\text{loc}}.} \quad (10.132)$$

For the mean universal first-passage time Θ , Dynkin's equation $\mathcal{L}_T \Theta = -1$ therefore gives

$$\boxed{\mathcal{L}_{\text{loc}} \Theta = -\frac{1}{N}.} \quad (10.133)$$

In spherical notation this may be written

$$\boxed{\frac{1}{A^{3/2} r^2} \frac{d}{dr} \left[r^2 A^{1/2} D_{\text{loc}} \frac{d\Theta}{dr} \right] = -\frac{1}{N},} \quad (10.134)$$

for the corresponding isotropic radial representation, with regularity at the centre and the chosen absorbing boundary condition at the exit surface.

N-OPT-01.1 — WEIGHTED-CADENCE THEOREM

On a frozen background with the same g_{ij} and D_{loc} , let Θ_N solve Eq. (10.133) and let $\Theta_{N=1}$ solve the same boundary-value problem with N replaced only by unity.

Then

$$\frac{\Theta_N(0)}{\Theta_{N=1}(0)} = \left\langle \frac{1}{N} \right\rangle_{K dV_{\text{prop}}},$$

and therefore

$$\min \frac{1}{N} \leq \frac{\Theta_N(0)}{\Theta_{N=1}(0)} \leq \max \frac{1}{N}.$$

The weighting kernel K is positive. The theorem isolates the cadence contribution; it does not compare two complete stellar models because g_{ij} and D_{loc} are deliberately held fixed.

III. Separation of physical roles

The result fixes three distinct inputs:

- D_{loc} belongs to **matter microphysics**: opacity, scattering, absorption, emission and the statistics of the local spatial step;
- g_{ij} belongs to the **relational spatial structure**: it determines the geometry in which the step is taken;
- N is the **cadence field**: it converts the rate of the local process to the universal line T through Eq. (10.131).

THE SENTENCE THAT SURVIVED THE AUDIT

Matter determines the statistics of the photon's step; State $\mathcal{A}$ determines the cadence of that step and the spatial structure in which it is taken.

No new optical coefficient is introduced by Eq. (10.133). In particular, the optical index derived from the eikonal sector is not re-labelled as a diffusion coefficient and N is not hidden inside D_{loc} . This prevents double counting between material transport and cadence conversion.

IV. Domain and falsification — N-OPT-02

The theorem is exact within the frozen-background Fick assumptions just stated. It does not yet compute a real stellar escape time. N-OPT-02 must test an explicit microscopic material sector and determine whether the same separation survives when the medium is dense, dispersive,

anisotropic or moving; whether $N(r)$ and $g_{ij}(r)$ can be obtained from the same admissible stellar solution; and whether D_{loc} contains no hidden conversion to T .

FALSIFICATION FRONTIER

If an admissible microscopic calculation requires another time-changed generator, a new fitted coefficient, or an additional cadence-dependent term solely to preserve Eq. (10.133), then H-OPT-01 / N-OPT-01.1 is modified or withdrawn rather than repaired. The result is not an astrophysical calibration and not an experimental validation of the RM ontology.

Current technical concordance received by v4.7.0

Pillar II remains the primary technical authority for the local gravitational, optical, canonical and strong-field derivations carried in this volume. Later narrow notes govern only the branches they explicitly test or replace:

- **Pillar I v5.3.0 — The Missing State of the Universe:** methodological authority for non-repair, separation of physical prerogatives and the current thermal/bulk concordance.
- **Pillar III v7.4.0:** current synthesis authority for the active architecture and historical branch register.
- **Pillar IV v7.2.0:** constitutional authority for ontology, laws, active postulates and the Exclusion Interaction.
- **The Uncollapsing Universe: Constitutive Exclusion and Homogeneous Bulk Closure in Relativity of Matter, v4.0.0:** narrow authority for GHC–1, GEC–1, HBR–1, the thermal/curvature extension and the Maria-bound status; DOI 10.5281/zenodo.22791787.
- **The Exclusion Interaction in Relativity of Matter, v1.0.0:** canonical EI–1 statement; DOI 10.5281/zenodo.22793642.
- **Geometry is Not Enough: Mechanical Constraints on Off-Center Cosmic Observables, v5.1.0:** current mechanical concordance for the reduced $U = 0$ matched-birth result, formation-cost separation, Core mechanics and stationary redshift transfer.
- **RM — Analysis of Dirac Canonical Structure under P–CIN:** local canonical health audit received here as P–CIN–DIRAC; it does not replace the open P–CIN–FIELD programme.

No later note is allowed to promote a conditional closure into a theorem by citation alone. Conversely, when a later action-level audit explicitly kills a provisional branch, the older branch remains historical rather than active.

v4.7.0 Consolidated Status Note

At the close of v4.7.0, Pillar II has applied its own non-repair rule. HCR-1/HCR-2 remain in the document as a historical calculation but no longer define the active homogeneous branch. The reduced zero mode is governed by GHC-1, GEC-1 and HBR-1 under the declared matter-radiation ledger. The thermal/curvature equation extends this branch and reduces to HBR-1 when $U = 0$; it does not rename the historical/global-elastic ε_z reservoir as the stellar thermal stock $\varepsilon_{\mathcal{E}}^{\text{th}}$.

Thermal-radiative loading is primary in the active stellar interpretation. Redshift remains a genuine propagation observable and a possible secondary energetic contribution, but no percentage, sign or receiver is assigned without derivation. The Maria bound is conditional on UC-1, and the bridge from approach to that bound to $EM \rightarrow \mathcal{E}$ remains a working constitutive bridge rather than a microscopic theorem. Boundary production is kept separate from the deep-bulk rate, and the unit formation-cost identification is conditional rather than universal.

The local strong-field programme gains one additional result: the P-CIN trace-free kinetic radical has regular momentum inversion, a positive convex Hamiltonian, a positive Hessian and a non-kinetic lapse. This closes the local kinetic-health audit only. The full field brackets, coupling to trace/curvature/matter and hyperbolicity remain open; no full two-polarisation theorem is inferred from the pointwise audit alone.

All frozen local results that do not depend on HCR remain unchanged: CLG-1, the Poisson and pair-energy limits, the N- $\mathcal{E}$ local constraint audits, the conditional PPN sector, the v28 stellar record, the CA-1 cross-method branch existence result and N-OPT-01.1.

v4.6.0 Consolidated Status Note

HISTORICAL STATUS AFTER v4.7.0

This note is retained as the dated closing status of v4.6.0. Its HCR-1/HCR-2 homogeneous branch is no longer active: v4.7.0 applies the non-repair clause and replaces that branch with the HBR homogeneous closure and its conditional thermal/curvature extension. The local and stellar statements below remain part of the historical audit according to their own statuses.

At the close of v4.6.0, the new homogeneous result has a deliberately tiered status. LF-09 is the constitutional Exclusion Interaction. HCR-2 is a candidate hyperelastic realisation of that law in the homogeneous deformation channel. HCR-1 is the provisional rate relation that defines the branch. Conditional on HCR-1/HCR-2, a finite positive State- $\mathcal{E}$ pressure produces a sub-capacity equilibrium $e_* < e_{\text{cap}}$; the equilibrium is locally attractive when $p'_{\mathcal{E}}(e_*) < \varepsilon'_z(e_*)$; a slowly moving equilibrium carries the finite-lag bound of HCR-5c; and exact $r = 0$ implies that the entire manifested-volume growth is boundary production, Eq. (HCR-6b).

None of these statements is advertised as an observational proof of LF-09. The decisive open task is now sharper: derive the global State- $\mathcal{E}$ action, its homogeneous pressure and Hamiltonian constraint, and test whether that common action reproduces HCR-1 without a fitted coefficient or sign repair. If it does not, the HCR branch is replaced. The old V2-style susceptibility and the rejected quadratic homogeneous kinetic repair are not reactivated.

The local and stellar results of v4.5.0 remain unchanged. In particular, no v28 coordinate, CA-1 result, PPN coefficient, local degree count, constitutive length or photonic-cadence theorem is retuned by the homogeneous revision.

v4.5.0 Consolidated Status Note

At the close of v4.5.0, two post-v4.4.1 results have been received. First, the regular $Q_c < 0$ stellar solution at $p_c = 0.5028$ is reproduced by DOP853 shooting and independent two-domain collocation; its branch existence and smooth $Q = 0$ crossing are therefore cross-method numerical results at that test point. Second, N-OPT-01.1 fixes the cadence contribution to a local Fick first-passage problem as the Dynkin cost $1/N$ on a frozen background. Neither result supplies a physical neutron-star calibration, radial stability, a complete Black-Core wall, the OCH-1 selector from one action, the full microscopic matter audit D-Æ-04M, or an observational validation.

The non-repair rule remains unchanged: dated calculations are not silently overwritten, and no new coefficient is admitted to save a branch after seeing the result.

Scientific responsibility

Pierre Benaros is the sole author and assumes full responsibility for all postulates, calculations, interpretations and claims in this document.