

Can an Off-Center Cosmic Position Be Observed from Purely Spatial Geometry?

Global elasticity, boundary production, and radial mechanical compatibility

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A spherical cosmic configuration fixes the sky axis of an off-center observer, while the amplitude requires a mechanically admissible field. We study this distinction in a spatial Relativity of Matter model with a global elastic reservoir, an invariant background, and a finite state-E Core. Exact variation gives a mean-cadence term in a screened scalar constraint; the static tensor equation excludes a nonzero gradient in the frozen homogeneous isotropic reduction. For boundary production, preserving both homogeneous constraints in the reduced $U = 0$ homogeneous branch with its declared material balances requires the new volume to inherit the global volume ratio, under a single uniform front and no reference remodeling. The formation-energy balance separates stored density, global reference work, and conversion cost. We derive radial dynamical equations with independent principal stresses, the canonically normalized momentum constraint, and the acceleration term in the bulk-side boundary traction. Exact constant-density static interiors and interface identities give criteria for the Core flux sign; density alone does not select it. These compatibility conditions connect boundary production, Core mechanics, and the observer's cadence gradient without using observed sky amplitudes, directions, or cosmological calibration.

I. QUESTION AND SCOPE

Spherical symmetry fixes one sky axis for an off-center observer. A nonzero amplitude requires a field whose sources, evolution, and boundary conditions are mutually compatible. This paper studies that mechanical problem in Relativity of Matter (RM), extending the geometric and global-reservoir analysis presented in Version 4 [1]. The frozen Version-2.2 axis and local-amplitude statements are reproduced in Sec. II.

The field $N > 0$ describes local cadence through $d\tau_{\text{cad}} = NdT$, where T is the universal open time parameter and τ_{cad} counts local physical processes. All geometry here is three-dimensional. The electromagnetic law and minimal gravitational action are stated below [2]. A finite-capacity restriction bounds admissible deformation in a constitutive channel; it does not by itself select an energy functional. The inhomogeneous global reservoir is an additional constitutive postulate. Section III displays local and global extensions explicitly, so this distinction requires no unpublished constitutive result.

The new results concern boundary production and radial dynamics. We derive reference-volume transport, show how simultaneous preservation of two homogeneous constraints selects a matched birth state under specified balances, and separate formation cost from deposited density. We then derive the radial equations with anisotropic stresses and the dynamical term in the inner bulk traction. Exact static interiors supply sign tests and analytical limits. The proper density u_E of state E remains a symbolic constitutive input.

The calculation distinguishes variation on a fixed instantaneous domain, physical boundary transport, and

matching to a Core or conversion layer. Treating these as the same operation would omit reference work or impose static traction on a dynamical configuration. The homogeneous companion note [3] supplies the reduced $U = 0$ balances used here and their later thermal extension; this paper invokes only the reduced branch where stated rather than presuming it for an arbitrary growing domain. No measured sky direction, dipole amplitude, expansion parameter, or catalogue enters the derivation. Table I collects the principal results and their hypotheses.

II. GEOMETRY, AXIS, AND LOCAL AMPLITUDE

Let (Σ, g) be a Riemannian spatial slice, O the physical center, and M an observer at geodesic radius r_o . Distinguish the Core, tracer support and manifested boundary:

$$R_C < r_o < R_{\text{tr}} \leq R_E. \quad (1)$$

The outward radial unit vector at M is $\hat{e}_r$; the direction toward the center is $-\hat{e}_r$. Radial formulas are used away from the center and the cut locus. Throughout, S denotes areal radius and $\mathcal{R}$ spatial scalar curvature; the boundary radii in Eq. (1) are geodesic radii.

a. Exact axisymmetry. Suppose the complete physical configuration, including its propagation laws, boundaries and relevant history, is invariant under rotations about O . An observer at rest, or moving along OM , is fixed by the rotations about OM . Covariance of the observation map therefore preserves this $SO(2)$ stabilizer. A scalar sky map satisfies

$$\mathcal{A}(R_\phi \hat{n}) = \mathcal{A}(\hat{n}), \quad a_{\ell m} = 0 \quad (m \neq 0) \quad (2)$$

in the aligned frame. Every nonzero dipole vector extracted covariantly from this configuration is parallel or

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antiparallel to $\hat{e}_r$. The statement is exact at arbitrary depth and does not fix the dipole's sign or magnitude. Vector and tensor maps transform with their appropriate representations; higher multipoles are not themselves dipole vectors.

b. Scalar line integrals. For a smooth scalar F and a unit-speed geodesic $\gamma_{\hat{n}}$ from M , define

$$\mathcal{O}_F(s, \hat{n}) = \int_0^s F(\gamma_{\hat{n}}(u)) du, \quad (3)$$

$$\mathcal{O}_F^- = \frac{1}{2}[\mathcal{O}_F(s, \hat{n}) - \mathcal{O}_F(s, -\hat{n})]. \quad (4)$$

Taylor expansion along the geodesic and integration give

$$\mathcal{O}_F^- = \frac{s^2}{2} \mathbf{D}F(M) \cdot \hat{n} + O(s^4). \quad (5)$$

For radial $F = f(r)$, $\mathbf{D}F = f'(r)\hat{e}_r$. If $\mathbf{D}F(M) = 0$, only this leading s^2 term is forced to vanish; higher odd contributions may survive. This is the Version-2.2 amplitude lock.

For counts with a smooth effective spatial density ν and geodesic-polar volume Jacobian J , the cumulative count is $\int_0^s \nu J du$. Since $J = u^2[1 - \text{Ric}(\hat{n}, \hat{n})u^2/6 + O(u^3)]$, its leading odd part and relative dipole amplitude are

$$\left(\frac{dN}{d\Omega}\right)^- = \frac{s^4}{4} \mathbf{D}\nu(M) \cdot \hat{n} + O(s^6), \quad (6)$$

$$\mathcal{D}_N(s) = \frac{3s}{4} |\mathbf{D} \ln \nu(M)| + O(s^3). \quad (7)$$

The relative amplitude in Eq. (7), including its $O(s^3)$ remainder, assumes $\nu(M) > 0$. These are local depth expansions, requiring s small compared with variation and curvature scales. A deep catalogue requires its full selection and propagation integrals. Neither equation identifies ν with N .

III. CONSTITUTIVE INPUT AND ITS VARIATION

A. Two extensions of the homogeneous reservoir

The homogeneous RM reservoir is written as

$$S_z = - \int dT N_0 V \varepsilon_z(e). \quad (8)$$

In the companion note, e is defined using a material element and its reference volume [3]. Homogeneous data alone cannot distinguish the local extension

$$H_z^A = \int_{\Omega} N \varepsilon_z(e(x)) dV_g, \quad e(x) = \ln \frac{\sqrt{g}(x)}{\sqrt{g_{\text{ref}}(x)}}, \quad (9)$$

from a global extension. The reference in Eq. (9) must be a specified material density transforming with the same weight as $\sqrt{g}$. Its physical and canonical status is additional input.

We adopt the following *Global Elasticity Postulate* (GEP) for the branch studied in this version:

$$\begin{aligned} H_z^B[N, g] &= \varepsilon_z(e) \int_{\Omega} N dV_g, \\ e &= \ln(V/V_{\text{ref}}), \quad V = \int_{\Omega} dV_g, \\ \bar{N} &= \frac{1}{V} \int_{\Omega} N dV_g. \end{aligned} \quad (10)$$

The domain Ω is the physical domain assigned to this reservoir; its relation to the Core and any distinct boundary phases must be specified in a complete model. The reference volume is positive. Both Ω and V_{ref} are held fixed during the following variations.

The GEP makes elastic response depend on the state of the whole specified body. The existence of one physical body does not, by itself, imply Eq. (10): ordinary local material energies and other nonlocal functionals remain logically possible. Nor does the statement that the invariant background is not stretched by a local orbit select a unique law for the distinct reservoir ε_z .

B. Exact functional derivatives

At fixed metric, e is independent of N , and

$$\frac{\delta H_z^B}{\delta N(x)} = \sqrt{g}(x) \varepsilon_z(e). \quad (11)$$

For $\delta g_{ij} = \omega g_{ij}$ in three dimensions,

$$\delta dV_g = \frac{3}{2} \omega dV_g, \quad (12)$$

$$\delta e = \frac{3}{2V} \int_{\Omega} \omega dV_g. \quad (13)$$

Writing $I = V\bar{N}$, the product rule gives $\delta H_z^B = \varepsilon'_z I \delta e + \varepsilon_z \delta I$ and hence

$$\boxed{\frac{\delta H_z^B}{\delta \omega(x)} = \frac{3}{2} \sqrt{g} [\varepsilon_z N(x) + \varepsilon'_z \bar{N}].} \quad (14)$$

The second term is the exact global contribution. Replacing $\bar{N}$ by $N(x)$ before variation would erase it. By comparison,

$$\frac{\delta H_z^A}{\delta \omega(x)} = \frac{3}{2} \sqrt{g} N(x) [\varepsilon_z + \varepsilon'_z](e(x)). \quad (15)$$

The extensions coincide on the homogeneous mode but differ off it.

The global functional contains no new cadence velocity or independent $\mathbf{D}N$ energy. For spatial relabellings preserving the specified domain, both integrals in Eq. (10) are invariant. This observation does not replace a full constraint-preservation and boundary analysis of the extended theory.

IV. THE SCALAR CADENCE CONSTRAINT

A. Local ingredients, including the invariant background

Write $\mathcal{R} = {}^{(3)}R[g]$ and $P = c_{\mathbb{E}}^4/(16\pi G) > 0$. The frozen RM Hamiltonian density has gravitational part

$$\mathcal{H}_g = \frac{c_{\mathbb{E}}^2}{P\sqrt{g}}\pi^{ij}\pi_{ij} - P\sqrt{g}\mathcal{R}, \quad (16)$$

where π^{ij} is trace-free on the constrained branch [2]. The canonical conformal generator $\int \omega g_{ij}\pi^{ij}d^3x$ sends $\delta g_{ij} = \omega g_{ij}$ and $\delta\pi^{ij} = -\omega\pi^{ij}$; it is this paired transformation that gives the kinetic coefficient in the conformal constraint. For the local matter/radiation sector let $\rho_{m\gamma}$ and $p_{m\gamma}$ denote energy density and mean pressure. We assume its fixed-matter-canonical-variable conformal variation is $-\frac{3}{2}\sqrt{g}Np_{m\gamma}$, with no additional metric-velocity contributions.

Retain the invariant background of the homogeneous closure, with density u_0 and response $p_0 = -u_0$ [3]. Extending that same interface here gives

$$\frac{c_{\mathbb{E}}^2}{P\sqrt{g}}\pi^{ij}\pi_{ij} - P\sqrt{g}\mathcal{R} + \sqrt{g}(\rho_{m\gamma} + u_0 + \varepsilon_z) = 0. \quad (17)$$

The conformal constraint $\mathcal{C} = -\delta H/\delta\omega = 0$ is

$$0 = \frac{3Nc_{\mathbb{E}}^2}{2P\sqrt{g}}\pi^{ij}\pi_{ij} + \frac{P}{2}N\sqrt{g}\mathcal{R} - 2P\sqrt{g}D^2N + \frac{3}{2}\sqrt{g}\{N(p_{m\gamma} - u_0 - \varepsilon_z) - \varepsilon'_z\tilde{N}\}. \quad (18)$$

Omitting u_0 and its pressure would fail to recover the quoted homogeneous closure.

B. Global response and the local comparison

Eliminating $\mathcal{R}$ between Eqs. (17) and (18) gives

$$D^2N = \mathcal{Q}_{\text{loc}}N + \alpha_z(N - \tilde{N}), \quad (19)$$

where

$$\begin{aligned} \alpha_z &= \frac{3\varepsilon'_z}{4P}, \\ \mathcal{Q}_{\text{loc}} &= \frac{c_{\mathbb{E}}^2}{P^2g}\pi^{ij}\pi_{ij} \\ &\quad + \frac{\rho_{m\gamma} + 3p_{m\gamma} - 2u_0 - 2\varepsilon_z - 3\varepsilon'_z}{4P}. \end{aligned} \quad (20)$$

All reservoir quantities in this expression are evaluated at the global e . Equivalently its effective pressure in the conformal constraint is

$$p_z^{\text{eff}}(x) = -\varepsilon_z - \varepsilon'_z \frac{\tilde{N}}{N(x)}. \quad (22)$$

For model A, the explicit mean term is absent and the corresponding local $e(x)$ enters $\mathcal{Q}$. That local equation is harmonic only where its full source vanishes; locality alone does not exclude other characteristic lengths.

On the isotropic homogeneous mode, $\pi^{ij} = 0$, $N = \tilde{N} = N_0$, cold matter has $p_m = 0$, and isotropic radiation has $p_\gamma = u_\gamma/3$. Equations (17)–(21) reduce to

$$P\mathcal{R} = \rho_m + u_\gamma + u_0 + \varepsilon_z \equiv \rho_{\text{tot}}, \quad (23)$$

$$\rho_m + 2u_\gamma - 2u_0 = 2\varepsilon_z + 3\varepsilon'_z. \quad (24)$$

Thus $\mathcal{Q}_{\text{loc}}^{(0)} = 0$ on that background. This identity cannot be imposed on every inhomogeneous region without checking the other field equations.

For $n = N - \tilde{N}$, the definition of the mean imposes

$$\int_{\Omega} n dV_g = 0. \quad (25)$$

At a fixed slice, $\tilde{N}$ has no spatial derivative. With $\varepsilon'_z > 0$ and $\mathcal{Q}_{\text{loc}} = \mathcal{Q}^{(1)}$ relative to the equilibrium background, Eq. (19) becomes

$$(D^2 - \lambda_z^{-2})n = \mathcal{Q}^{(1)}(\tilde{N} + n), \quad \lambda_z^2 = \frac{4P}{3\varepsilon'_z}. \quad (26)$$

The sign is that of a screened elliptic operator, using $D^2 = D_i D^i$. The coefficients have dimensions $[P] = \text{J m}^{-1}$, $[\varepsilon'_z] = \text{J m}^{-3}$, and $[\alpha_z] = \text{m}^{-2}$. If geometry, matter and $\mathcal{Q}^{(1)}$ respond to N , this remains part of a coupled system rather than a closed linear equation.

V. A BACKGROUND LOWER BOUND ON THE SCREENING LENGTH

For the illustrative finite-capacity law

$$\varepsilon_z(e) = k_z \frac{e}{e_{\text{cap}} - e}, \quad k_z > 0, \quad 0 \leq e < e_{\text{cap}}, \quad (27)$$

one has $\varepsilon'_z = k_z e_{\text{cap}}/(e_{\text{cap}} - e)^2 > 0$. The following bound needs only positivity, not this analytic form. Assume the homogeneous component densities $\rho_m, u_\gamma, u_0, \varepsilon_z$ are nonnegative and satisfy Eq. (24). Then

$$0 < 3\varepsilon'_z = \rho_m + 2u_\gamma - 2u_0 - 2\varepsilon_z \leq \rho_m + 2u_\gamma. \quad (28)$$

The homogeneous constant-curvature representative has

$$\mathcal{R} = 6\kappa > 0, \quad \mathfrak{a}^2 = \kappa^{-1} = \frac{6P}{\rho_{\text{tot}}}. \quad (29)$$

Consequently,

$$\begin{aligned} \frac{\lambda_z^2}{\mathfrak{a}^2} &= \frac{2\rho_{\text{tot}}}{9\varepsilon'_z} \\ &\geq \frac{2\rho_{\text{tot}}}{3(\rho_m + 2u_\gamma)} \geq \frac{2}{3(1 + f_\gamma)}, \end{aligned} \quad (30)$$

$$f_\gamma = \frac{u_\gamma}{\rho_m + u_\gamma} \in [0, 1]. \quad (31)$$

In particular,

$$\lambda_z \geq \frac{\mathfrak{a}}{\sqrt{3}}; \quad u_\gamma = 0 \implies \lambda_z \geq \sqrt{\frac{2}{3}} \mathfrak{a}. \quad (32)$$

This is a lower bound relative to a background curvature radius. It supplies no upper bound, no measured length, and no identification of $\mathfrak{a}$ with $R_{\mathcal{E}}$. It does not demonstrate confinement of the Core field or a universal attenuation at the observer. In particular, the positive-density homogeneous branch is not spatially flat.

VI. FINITE DOMAINS, CORE DATA, AND BOUNDARY RESPONSE

A. A flux identity survives screening

A general spherical spatial metric in proper radial distance is

$$d\ell^2 = dr^2 + S(r)^2 d\Omega^2, \quad (33)$$

where S is the areal radius. Equation (19) implies

$$\begin{aligned} S(r)^2 N'(r) &= C_C \\ &+ \int_{R_C}^r S^2(\mathcal{Q}_{\text{loc}} N + \alpha_z n) du, \quad (34) \\ C_C &\equiv S(R_C)^2 N'(R_C^+). \end{aligned}$$

The integrand is evaluated at u . The scalar equation is used only where its constitutive assumptions apply. The Core enters the exterior equation through matching data. Sections VIII–X derive those data for explicit interior branches. No regularity condition at $r = 0$ is imposed on an exterior equation.

Gauss' divergence identity is not lost. What fails, in general, is the reduction of the bulk flux to a radius-independent Core datum: $\alpha_z n$ participates throughout the bulk. Boundary conditions can also affect N , the mean and the matching data themselves. A vanishing boundary contribution in a particular solution would not, by itself, refute the GEP.

In a source-free local-reservoir comparison, the integral in Eq. (34) is zero and $N' = C_C/S^2$. At a fixed positive cadence calibration $N_\star$, one may define

$$\mathfrak{M}_C^{\text{flux}} = \frac{c_{\mathcal{E}}^2 C_C}{G N_\star}. \quad (35)$$

This is a calibrated junction parameter. Identifying it with a physical active mass requires the Core equations and any interface stresses. If the same local scalar law holds inside a static regular Core, with no singular layer, it reduces to

$$\mathfrak{M}_C^{\text{flux}} = \frac{1}{c_{\mathcal{E}}^2 N_\star} \int_C (\rho + 3p) N dV_g. \quad (36)$$

The ordinary enclosed mass is recovered only in the appropriate weak-field, negligible-pressure limit. These conditions are not presumed for the Cosmic Core.

B. A curved radial benchmark

For a prescribed constant-curvature metric, let

$$S_\kappa(r) = \begin{cases} \sin(\sqrt{\kappa}r)/\sqrt{\kappa}, & \kappa > 0, \\ r, & \kappa = 0, \\ \sinh(\sqrt{-\kappa}r)/\sqrt{-\kappa}, & \kappa < 0. \end{cases} \quad (37)$$

On an annulus with $\mathcal{Q}^{(1)} = 0$, setting $u = S_\kappa n$ reduces the scalar equation to

$$u'' - (\lambda_z^{-2} - \kappa)u = 0. \quad (38)$$

Thus the flat-annulus solution is

$$n(r) = \frac{Ae^{-r/\lambda_z} + Be^{r/\lambda_z}}{r}. \quad (39)$$

Keeping only the decaying branch presupposes an additional outer condition, often one at infinity that is unavailable for a finite cosmic body. At a finite material surface, traction or matching data must instead be specified using the radial equation underlying Sec. XI C, with the outer normal orientation and any interface balances. The inner-boundary sign in Eq. (85) cannot be copied unchanged. Appendix A therefore retains both boundary data for any sign of $\lambda_z^{-2} - \kappa$.

This is a benchmark for the *scalar operator on a prescribed geometry*. The next section shows why it cannot automatically be promoted to a static solution of the coupled RM equations. Independently, its boundary values and $\bar{N}$ must satisfy Eq. (25), including all regions entering the average. A strictly one-sign n over all Ω cannot satisfy that condition.

C. Integral compatibility and the zero-gradient branch

If Eq. (19) holds throughout the reservoir domain, with all interface terms accounted for and uniform α_z on the slice, integration gives

$$\int_{\partial\Omega} \partial_\nu N dS = \int_\Omega \mathcal{Q}_{\text{loc}} N dV_g. \quad (40)$$

The mean term cancels globally. If the equation holds only in a bulk subdomain, its integral over that subdomain does not in general cancel; the Core and boundary matching terms remain.

For the particular regular compact case $\mathcal{Q}_{\text{loc}} = 0$ throughout Ω , with no boundary or with $\partial_\nu N = 0$, multiplying by n and integrating by parts yields

$$\int_\Omega (|DN|^2 + \lambda_z^{-2} n^2) dV_g = 0. \quad (41)$$

Hence $N = \bar{N}$. Global coupling does not create an inhomogeneity in an otherwise unforced configuration. A source or suitable boundary data are needed, and the full field equations must still admit them.

VII. STATIC TENSOR COMPATIBILITY: AN OBSTRUCTION

The scalar equation is a necessary condition. For a strictly static configuration with vanishing shift and $\pi^{ij} = 0$, the gravitational potential is $-P \int N \sqrt{g} \mathcal{R} d^3x$. With compactly supported metric variations, its trace-free variation is proportional to

$$N (R_{ij} - \frac{1}{3} \mathcal{R} g_{ij}) - (D_i D_j N - \frac{1}{3} g_{ij} D^2 N). \quad (42)$$

The global reservoir has only an isotropic metric response. If all other stresses are also isotropic, stationarity therefore requires

$$(D_i D_j N)^{\text{TF}} = N R_{ij}^{\text{TF}}. \quad (43)$$

This spatial variational identity is related to the usual static-potential identities in Riemannian geometry [4]; no spacetime field equation is imported.

Taking the divergence, using the commutator on a scalar gradient and the contracted Bianchi identity, gives

$$D_i (D^2 N) + \frac{\mathcal{R}}{2} D_i N - \frac{N}{4} D_i \mathcal{R} = 0. \quad (44)$$

a. Static homogeneous-bulk obstruction. In a connected region assume: strict stationarity, vanishing trace-free momentum, isotropic stresses, constant scalar curvature $\mathcal{R}$, constant positive α_z , and $\mathcal{Q}_{\text{loc}} = 0$. Combining Eq. (44) with $D^2 N = \alpha_z (N - \bar{N})$ gives

$$(\alpha_z + \mathcal{R}/2) D_i N = 0. \quad (45)$$

For $\alpha_z + \mathcal{R}/2 > 0$, and in particular on the positive-curvature homogeneous branch, $DN = 0$. Substitution into the scalar equation then gives $N = \bar{N}$ in that region. The result requires constant *scalar* curvature; it does not require constant sectional curvature.

Thus a nonzero static Core-induced profile cannot be obtained by freezing every homogeneous isotropic bulk ingredient and solving only a Yukawa equation. A physical continuation must allow appropriate spatially varying loads/curvature, anisotropic stresses or time dependence, and solve the corresponding equations together. Their role must be derived, rather than inserted solely to preserve a desired dipole. The obstruction does not invalidate the variational contribution in Eq. (14); it identifies a restricted completion in which the gradient is forced to zero.

The local principal part of the scalar constraint remains elliptic, but the added global dependence and the choice of boundary conditions can alter its kernel. A complete global Dirac analysis and a stability analysis are not replaced by the local degree count of the frozen branch.

VIII. CORE MECHANICS AND THE ACTIVE SIGN

The RM state inventory distinguishes localized matter m , propagating EM, the continuous transmissive state

$\mathbb{E}$, and state E of maximal concentration and cohesion [1]. The Core is assigned to E. A saturation density and a mechanical response are different constitutive data: naming the former does not select the latter. Write its uniform proper energy density as $u_E > 0$. No density anchor is used to determine u_E in this paper; the results hold for symbolic u_E .

We use the two-derivative gravitational branch underlying Eqs. (17) and (43). Its material interface is static and isotropic, with no interior conversion or additional volume force. This defines the domain of the following calculation. Applying it to the nonlinear constitutive-curvature sector near saturation requires that sector's own field equations [2].

A. Intensive energy and constrained saturation

For an energy law $\mathcal{U}_I(V) = u_E V$ under volume variations at fixed u_E , the mechanical pressure is

$$p_I = - \left(\frac{\partial \mathcal{U}_I}{\partial V} \right)_{u_E} = -u_E, \quad u_E + 3p_I = -2u_E. \quad (46)$$

This intensive law gives tension. By itself it has zero bulk modulus, $-V \partial p_I / \partial V = 0$; an incompressibility constraint is a distinct mechanical assumption.

Let $J = dV_g / dV_{\text{ref}}$ be the local volume ratio relative to a fixed material reference measure. An explicit constrained realization is

$$H_E = \int_C N_E [u_E J - \Pi_E (J - 1)] dV_{\text{ref}}. \quad (47)$$

Variation of Π_E imposes $J = 1$. Varying the volume before enforcing that constraint gives

$$p_E = -u_E + \Pi_E. \quad (48)$$

At fixed N_E, Π_E , the coefficient of δJ in the energy variation is $N_E (u_E - \Pi_E) = -N_E p_E$. On the constraint the energy density is u_E for every multiplier value. Substitution of $J = 1$ before variation would erase the reaction pressure.

Equation (47) declares a constitutive realization, rather than selecting a unique microscopic action for E. A unilateral constraint $J \geq 1$ can impose $\Pi_E \geq 0$ and complementary slackness at saturation; positivity of that reaction alone does not fix the sign of $-2u_E + 3\Pi_E$. Shear or anisotropic stress requires additional material response.

B. A first integral and a volume-mass criterion

The material equilibrium equation in the static minimal branch is [2]

$$p'_E = -(u_E + p_E) \frac{N'_E}{N_E}. \quad (49)$$

For constant u_E , Eqs. (48) and (49) yield

$$\boxed{(\Pi_E N_E)' = 0, \quad \Pi_E N_E = C_E.} \quad (50)$$

At an interior boundary with pressure $p_{E,b}$ and cadence $N_b > 0$,

$$C_E = (u_E + p_{E,b})N_b, \quad p_E(r) = \frac{C_E}{N_E(r)} - u_E. \quad (51)$$

The integration constant is determined by the mechanical solution and its matching data. The intensive branch has $C_E = 0$.

The isotropic E contribution to the active volume mass is

$$\mathfrak{M}_E^{\text{vol}} = \frac{1}{c_{\mathcal{E}}^2 N_\star} \int_C (u_E + 3p_E) N_E dV_g. \quad (52)$$

With $V_C = \text{Vol}_g(C)$ and $\langle N_E \rangle_C = V_C^{-1} \int_C N_E dV_g$, the first integral gives

$$\boxed{\mathfrak{M}_E^{\text{vol}} = \frac{V_C}{c_{\mathcal{E}}^2 N_\star} [3C_E - 2u_E \langle N_E \rangle_C].} \quad (53)$$

The sign is positive precisely when $C_E > (2/3)u_E \langle N_E \rangle_C$, zero at equality, and negative below it. Neither a uniform density nor a positive reaction pressure alone selects this sign. The invariant background, elastic reservoir and interface layers are separate contributions when present. For anisotropic stresses, the local source becomes $u_E + p_r + 2p_t$ and Eq. (49) also changes.

IX. AN EXACT SPHERICAL BENCHMARK AND ITS JUNCTION

A. Solving all static interior equations

Consider a regular spherical interior with constant total density $u > 0$, isotropic pressure p , zero shift and vanishing momentum. Initially this is a single-sector test, $u = u_E$ and $p = p_E$. The full minimal static system is

$$P\mathcal{R} = u, \quad (54a)$$

$$D^2 N = \frac{u + 3p}{4P} N, \quad (54b)$$

$$(D_i D_j N)^{\text{TF}} = N R_{ij}^{\text{TF}}, \quad (54c)$$

$$p' = -(u + p)N'/N. \quad (54d)$$

The familiar constant-density static family [5] serves here as a consistency and junction benchmark, not as a new mathematical family or a unique cosmic interior. Its spatial equations are derived and checked within the stated RM branch.

Use proper radial distance, $d\ell^2 = dr^2 + S(r)^2 d\Omega^2$. The Hamiltonian constraint reads

$$-4\frac{S''}{S} + 2\frac{1 - S'^2}{S^2} = 6\kappa_C, \quad \kappa_C = \frac{u}{6P} > 0. \quad (55)$$

On an increasing-areal-radius branch its first integral is $S'^2 = 1 - \kappa_C S^2 - C_g/S$. Regularity at the center sets $C_g = 0$, and hence

$$S(r) = \frac{\sin(\sqrt{\kappa_C} r)}{\sqrt{\kappa_C}}, \quad y(r) = \cos(\sqrt{\kappa_C} r). \quad (56)$$

The trace-free equation reduces to $N'' - (S'/S)N' = 0$. Thus

$$N = A + By, \quad p = u \left(\frac{2A}{3N} - 1 \right), \quad (57)$$

where the pressure follows from the scalar cadence equation and satisfies hydrostatic equilibrium. The Core curvature κ_C need not equal the homogeneous bulk curvature κ of Sec. V.

B. Boundary pressure selects the surface-flux sign

Restrict to $0 < \sqrt{\kappa_C} R_C < \pi/2$. Define $y_b = y(R_C) > 0$, $S_b = S(R_C)$, $N_b = N(R_C) > 0$, and $w_b = p_b/u$. The interior is

$$N(r) = \frac{N_b}{2} \left[3(1 + w_b) - (1 + 3w_b) \frac{y(r)}{y_b} \right], \quad (58)$$

$$p(r) = u \left[(1 + w_b) \frac{N_b}{N(r)} - 1 \right], \quad (59)$$

$$N'(r) = \frac{\kappa_C S(r) N_b}{2y_b} (1 + 3w_b). \quad (60)$$

Parameters are restricted to $N > 0$. If $w_b > -1/3$, the center gives the strict condition $3(1 + w_b)y_b > 1 + 3w_b$; if $w_b \leq -1/3$, the minimum cadence on this interval is the positive boundary value.

In the absence of a singular surface layer, the surface flux equals the total active volume integral and is

$$\boxed{\mathfrak{M}_C^{\text{flux}} = \frac{4\pi u S_b^3}{3c_{\mathcal{E}}^2} \frac{N_b}{N_\star y_b} (1 + 3w_b).} \quad (61)$$

This follows directly from $c_{\mathcal{E}}^2 S_b^2 N_b' / (GN_\star)$ and $\kappa_C = u/(6P)$. The areal-radius cube is an exact algebraic factor; the proper volume remains $4\pi \int_0^{R_C} S^2 dr$.

For the same u , R_C and N_b , with $y_b > 1/3$, three regular positive-cadence interiors give

Boundary ratio	Interior response	Flux sign
$w_b = -1$	$p = -u$, $N = N_b y / y_b$	negative
$w_b = -1/3$	$p = -u/3$, $N = N_b$	zero
$w_b = 0$	$p \geq 0$	positive

All four equations (54) hold for each case. These interiors establish explicitly that constant density does not determine the active sign. They also satisfy the tensor test of Sec. VII: the pressure varies as required and the homogeneous-bulk condition $\mathcal{Q}_{\text{loc}} = 0$ is not imposed.

C. A layer-free junction to the minimal vacuum

For a static minimal vacuum exterior, the canonical branch has $S'^2 = 1 - 2Gm_b/(c_{\mathcal{E}}^2 S)$ and N proportional to S' [2]. Continuity of S and S' at a layer-free boundary fixes its geometric mass parameter to

$$m_b = \frac{4\pi u S_b^3}{3c_{\mathcal{E}}^2} > 0. \quad (62)$$

The exterior cadence then obeys

$$\left. \frac{N'}{N} \right|_{b+} = \frac{\kappa_C S_b}{2y_b}. \quad (63)$$

Continuity of N and its flux, compared with Eq. (60), requires

$$p_b = 0. \quad (64)$$

With exterior calibration $N = N_* S'$, Eq. (61) reduces to $\mathfrak{M}_C^{\text{flux}} = m_b$. This also checks the mass normalization without a hidden factor.

An intensive interior $p = -u$ cannot meet this particular vacuum without a layer. A different exterior, an interface stress or evolution changes the junction problem. The cosmological RM bulk carries the sources in Eq. (19); it is not the vacuum of this benchmark. Surface matching therefore participates in selecting the physical branch.

X. THE GLOBAL RESERVOIR ACROSS THE CORE

Suppose the same GEP reservoir and invariant background extend throughout the Core, and E obeys Eq. (49) without an extra volume force. On a given slice define

$$b_0 = u_0 + \varepsilon_z, \quad a_0 = \varepsilon'_z \bar{N}, \quad u = u_E + b_0 > 0. \quad (65)$$

The total pressure supplied by the variations is

$$p_{\text{tot}} = p_E - b_0 - \frac{a_0}{N}, \quad (u + p_{\text{tot}})N = C_E - a_0. \quad (66)$$

Since a_0, b_0 are spatial constants, the total pressure also satisfies $p'_{\text{tot}} = -(u + p_{\text{tot}})N'/N$. The four equations (54) thus apply locally to (u, p_{tot}) . Equations (58)–(61) remain an exact interior representative, with $w_b = p_{\text{tot},b}/u$ and the positivity conditions stated above.

Its surface-flux sign is

$$\boxed{\text{sgn } N'(R_C) = \text{sgn} \left[u_E + 3p_{E,b} - 2(u_0 + \varepsilon_z) - 3\varepsilon'_z \frac{\bar{N}}{N_b} \right]}. \quad (67)$$

The intensive choice $p_E = -u_E$ gives a negative total source when $u > 0$ and $\varepsilon'_z > 0$. A positive E boundary

pressure alone does not guarantee the opposite sign: the background and reservoir contributions also enter.

This continuation is conditional on the reservoir occupying E with the same constitutive law. If E excludes that reservoir or changes its law, the corresponding interior and interface equations replace Eq. (66). In either case $\bar{N}$ is the actual mean on the reservoir domain, not a freely prescribed closing parameter.

Finally, even a selected surface-flux sign must be transported through the bulk. For a layer-free Core datum, or with any interface contribution already included in $\mathfrak{M}_C^{\text{flux}}$, Eq. (34) gives

$$S(r_o)^2 N'(r_o) = \frac{GN_*}{c_{\mathcal{E}}^2} \mathfrak{M}_C^{\text{flux}} + \int_{R_C}^{r_o} S^2 [\mathcal{Q}_{\text{loc}} N + \alpha_z (N - \bar{N})] dr. \quad (68)$$

Here $S(r)$ is the areal radius of the solved bulk. The E volume contribution, the Core surface flux and the observer's gradient are distinct, explicitly connected quantities. In particular, the last integral can change the relation between the surface sign and the sign at M .

A. Matching to the loaded cosmological bulk

For a static isotropic interface without a singular layer, continuity of the metric and cadence derivatives implies continuity of total radial pressure. At the inner bulk boundary,

$$p_{B,b} = p_{m\gamma,b} - b_0 - a_0/N_b. \quad (69)$$

For a regular uniform interior of total density u_C , write S for its areal radius and $y = S'$. Equation (60) gives

$$N'_{C,b} = \frac{S_b N_b}{12P y_b} (u_C + 3p_{C,b}). \quad (70)$$

If background and reservoir exclude E, $u_C = u_E$ and $p_{C,b} = p_{E,b} = p_{B,b}$. Then

$$\boxed{N'_b = \frac{S_b}{12P y_b} \{N_b [u_E + 3p_{m\gamma,b} - 3b_0] - 3a_0\}}. \quad (71)$$

If the same background and reservoir occupy E, $u_C = u_E + b_0$ and pressure continuity gives $p_{E,b} = p_{m\gamma,b}$. Thus

$$\boxed{N'_b = \frac{S_b}{12P y_b} \{N_b [u_E + 3p_{m\gamma,b} - 2b_0] - 3a_0\}}. \quad (72)$$

The geometries differ because u_C differs. With $S_b, y_b, N_b > 0$, each sign is fixed by its bracket, subject to positive interior cadence and an admissible E reaction.

Small potential at the observer does not imply uniform smallness down to the Core. Even in a Newtonian comparison,

$$\frac{G|\mathfrak{M}|}{c_{\mathcal{E}}^2 R_C} = \frac{r_o}{R_C} \frac{G|\mathfrak{M}|}{c_{\mathcal{E}}^2 r_o}. \quad (73)$$

Consequently smallness at r_o alone does not establish $\tilde{N}/N_b = 1 + O(\epsilon)$. Nor does it prove that the actual Core is in strong field: either conclusion requires its solved parameters and matched field.

XI. RADIAL DYNAMICS AND BULK-SIDE TRACTION

A. Action and canonical momentum

The minimal spatial gravitational action is

$$S_g = P \int dT d^3x \sqrt{g} \left[\frac{K_{ij} K^{ij} - K^2/3}{4Nc_{\mathcal{E}}^2} + N\mathcal{R} \right], \quad (74)$$

where $K_{ij} = \dot{g}_{ij} - \mathcal{L}_\beta g_{ij}$. The shift β is varied before setting it to zero. Matter has no additional metric-velocity terms, and its principal stresses p_r, p_t are defined by the canonical source variations. Radial transport does not in general preserve isotropic spatial stress.

For $d\ell^2 = A^2 dr^2 + S^2 d\Omega^2$ and zero shift, define

$$\begin{aligned} \zeta &= \dot{A}/A - \dot{S}/S, & \vartheta &= \zeta/N, \\ H_t &= \dot{S}/(NS), & D_\tau &= N^{-1} \partial_T. \end{aligned} \quad (75)$$

The last operator measures a rate per local cadence. Since $K_{\text{TF}}^{ij} K_{ij}^{\text{TF}} = 8\zeta^2/3$, only the trace-free rate contributes to this gravitational kinetic term. The canonical tensor density and its ordinary mixed tensor are

$$\pi^{ij} = \frac{P\sqrt{g}}{2Nc_{\mathcal{E}}^2} K_{\text{TF}}^{ij}, \quad (76)$$

$$\Pi^j_i := \pi^j_i / \sqrt{g} = \frac{P}{3c_{\mathcal{E}}^2} \text{diag}(2\vartheta, -\vartheta, -\vartheta). \quad (77)$$

Its radial divergence is

$$\begin{aligned} D_j \Pi^j_r &= (\Pi^r_r)' + 2 \frac{S'}{S} (\Pi^r_r - \Pi^\theta_\theta) \\ &= \frac{2P}{3c_{\mathcal{E}}^2} \left(\vartheta' + 3 \frac{S'}{S} \vartheta \right). \end{aligned} \quad (78)$$

The A'/A connection terms cancel. With the canonical convention $\mathcal{H}_i = -2D_j \pi^j_i + \mathcal{H}_i^m = 0$ [2] and $j_r = \mathcal{H}_r^m / \sqrt{g}$,

$$j_r = \frac{4P}{3c_{\mathcal{E}}^2} \left(\vartheta' + 3 \frac{S'}{S} \vartheta \right). \quad (79)$$

Thus $j_r = 0$ implies $S^3 \vartheta$ constant. Regularity sets that constant to zero on a region containing the center; a source-free annulus can have nonzero constant.

B. Evolution equations and their trace

Vary before imposing $A = 1, A' = 0$ on one slice, retaining $\dot{A}, \dot{A}$. This is an instantaneous proper-distance

gauge, not a gauge imposed for all T . Write $y = S'$ on that slice. The Hamiltonian constraint is

$$P\mathcal{R} = \rho_{\text{tot}} + \frac{2P}{3c_{\mathcal{E}}^2} \vartheta^2. \quad (80)$$

The radial and tangential metric equations are

$$\begin{aligned} \frac{y}{S} \frac{N'}{N} - \frac{1-y^2}{2S^2} - \frac{p_r}{4P} &= -\frac{\mathcal{B}_r}{3c_{\mathcal{E}}^2}, \\ N'' + \frac{y}{S} N' + N \frac{S''}{S} - \frac{N p_t}{2P} &= \frac{N}{3c_{\mathcal{E}}^2} (D_\tau \vartheta + 2\vartheta^2 + 3\vartheta H_t), \end{aligned} \quad (81)$$

where $\mathcal{B}_r = D_\tau \vartheta + \vartheta^2/2 + 3\vartheta H_t$. Appendix B gives both variations. Adding Eq. (82) to N times Eq. (81), with all terms on one side, gives

$$D^2 N = N \left[\frac{\mathcal{R}}{4} + \frac{p_r + 2p_t}{4P} + \frac{\vartheta^2}{2c_{\mathcal{E}}^2} \right]. \quad (83)$$

Both reservoir principal pressures contain $-b_0 - a_0/N$. Eliminating curvature with Eq. (80) yields

$$D^2 N = \mathcal{F}, \quad \mathcal{F} = N \left[\frac{\rho_{\text{tot}} + p_r + 2p_t}{4P} + \frac{2\vartheta^2}{3c_{\mathcal{E}}^2} \right]. \quad (84)$$

This recovers Eq. (19), including its kinetic coefficient, with $3p$ the spatial stress trace.

The static limit requires $\vartheta = D_\tau \vartheta = 0$. If only $\vartheta = 0$ at an instant, Eq. (81) instead fixes its acceleration from the static traction residual. That acceleration is not itself a matter velocity or the direction of a material current. Initial data must satisfy the Hamiltonian, cadence, and momentum constraints and the material equations.

C. Inner boundary traction

At the inner bulk boundary, $\nu_B = -\partial_r$ on the slice and $p_{r,B} = p_{r,m\gamma} - b_0 - a_0/N$. Multiplication of Eq. (81) by SN gives

$$\begin{aligned} y_b \partial_{\nu_B} N + \left[\frac{1-y_b^2}{2S_b} + \frac{S_b(p_{r,m\gamma,b} - b_0)}{4P} \right. \\ \left. - \frac{S_b \mathcal{B}_{r,b}}{3c_{\mathcal{E}}^2} \right] N_b &= \frac{S_b a_0}{4P}. \end{aligned} \quad (85)$$

All quantities are bulk-side values. For $y_b \neq 0$, resolved trace, geometry, mean, and dynamical data fix the normal derivative. With those quantities unknown this is a nonlocal traction relation, not an independent numerical Neumann datum. At $y_b = 0$, the undivided equation remains a compatibility condition.

The acceleration term has a minus sign. Setting $\vartheta = D_\tau \vartheta = 0$ gives the static Robin relation retaining the background and reservoir. A moving or converting material interface additionally obeys its own energy and momentum balances.

XII. INSTANTANEOUS FLUX AND A STATIC INTERFACE LAYER

On a proper-distance slice, $(S^2 N')' = S^2 \mathcal{F}$. For a regular Core with continuous S, N and a possible surface delta source j_Σ , integration gives

$$\partial_{\nu_B} N|_b = -\frac{1}{S_b^2} \int_0^{R_C} S^2 \mathcal{F}_C dr - j_\Sigma. \quad (86)$$

The sources must correspond to the actual interior branch. The identity also holds on a dynamical slice when its constraint and interface distributions apply. It supplies the normal derivative from resolved interior and layer data.

For a static layer in the minimal branch, let σ be energy per area and p_Σ tangential pressure per area, without radial delta stress. Define $[X] = X_B - X_C$ and $\langle X \rangle_\Sigma = (X_B + X_C)/2$. With the same P on both sides, singular curvature and scalar-source terms give

$$[S'] = -\frac{\sigma S_b}{4P}, \quad [N'] = \frac{N_b(\sigma + 2p_\Sigma)}{4P}. \quad (87)$$

Thus $j_\Sigma = N_b(\sigma + 2p_\Sigma)/(4P)$. The regular static radial equations on each side imply

$$[p_r] = -\sigma \left\langle \frac{N'}{N} \right\rangle_\Sigma + 2p_\Sigma \left\langle \frac{S'}{S} \right\rangle_\Sigma. \quad (88)$$

Indeed $p_r = 4P(S'/S)(N'/N) - 2P(1 - S'^2)/S^2$ and $[ab] = \langle a \rangle [b] + \langle b \rangle [a]$. A tension $\gamma > 0$ has $p_\Sigma = -\gamma$; its weak-field contribution gives $p_C - p_B = 2\gamma/S_b$.

For $N'_{C,b} > 0$, this static layer reverses the transmitted derivative precisely when

$$\sigma + 2p_\Sigma < -\frac{4P}{N_b} N'_{C,b}. \quad (89)$$

It also changes the traction and geometry, which must remain compatible. Moving layers require their own dynamical balances; Eq. (87) is the explicitly static realization.

XIII. BOUNDARY PRODUCTION AND THE REFERENCE STATE

A. Transport and birth ratio

Let θ be the volume-expansion rate of old material per unit T and q the normal production velocity relative to that material in the same time convention. Define

$$3H_V = V^{-1} \int_\Omega \theta dV_g, \quad \Gamma_\partial = \int_{\partial\Omega} q dA_g, \quad (90)$$

$$\dot{V} = 3H_V V + \Gamma_\partial, \quad 3g_\partial = \Gamma_\partial/V. \quad (91)$$

Here $q \geq 0$ on the production surface and other boundaries are material. In spherical symmetry $\Gamma_\partial = 4\pi S_\partial^2 q_\partial$ with areal radius S_∂ .

Let $J_b = dV_{\text{new}}/dV_{\text{ref,new}} > 0$ and $\bar{J} = V/V_{\text{ref}}$. Without remodeling of old reference material,

$$\dot{V}_{\text{ref}} = \int_{\partial\Omega} \frac{q}{J_b} dA_g. \quad (92)$$

Uniform birth gives

$$\dot{e} = 3H_V + 3g_\partial(1 - \bar{J}/J_b). \quad (93)$$

For $\Gamma_\partial > 0$,

$$\begin{aligned} \dot{e} = 3H_V &\iff \dot{V}_{\text{ref}}/V_{\text{ref}} = 3g_\partial \\ &\iff J_b = \bar{J}. \end{aligned} \quad (94)$$

This concerns a scalar volume ratio, not an unspecified strain tensor. Transport alone does not select it. Relaxed birth $J_b = 1$ instead contributes $3g_\partial(1 - e^e)$ to $\dot{e} - 3H_V$.

B. Simultaneous homogeneous constraint preservation in the reduced $U = 0$ branch

This subsection deliberately uses the reduced $U = 0$ homogeneous branch of the companion closure [3]. Its global elastic reservoir $\varepsilon_z(e)$ is not identified here with the stellar thermal stock $\varepsilon_{\mathcal{E}}^{\text{th}}$. Importing the enlarged thermal sector $U(X)$ into the growing-domain compatibility problem would require a separate rederivation of the constraint-preservation equations.

For the homogeneous old-material reduction, set $H = H_V$ and retain the published reduced balances [3]:

$$\dot{\rho}_m = -3H\rho_m - s_{\text{ex}}, \quad \dot{u}_\gamma = -4Hu_\gamma + s_{\text{ex}}, \quad (95)$$

$$\dot{u}_0 = 0, \quad \dot{\mathcal{R}} = -2H\mathcal{R}. \quad (96)$$

The exchange s_{ex} is internal to matter and radiation in this reduction. Set

$$B(e) = 2\varepsilon_z + 3\varepsilon'_z, \quad \delta = \dot{e} - 3H, \quad (97)$$

$$C_H = PR - \rho_m - u_\gamma - u_0 - \varepsilon_z. \quad (98)$$

The instantaneous constraints are $C_H = 0$ and $\rho_m + 2u_\gamma - 2u_0 = B(e)$. Differentiating gives

$$\begin{aligned} \dot{C}_H &= -2HPR + 3H\rho_m + 4Hu_\gamma - \varepsilon'_z \dot{e} \\ &= H(\rho_m + 2u_\gamma - 2u_0 - 2\varepsilon_z) - \varepsilon'_z \dot{e} \\ &= \boxed{-\varepsilon'_z \delta}. \end{aligned} \quad (99)$$

For $\varepsilon'_z > 0$, simultaneous preservation therefore requires $\delta = 0$. One nonzero uniform front without remodeling then has $J_b = \bar{J}$. This is a necessary compatibility condition within the declared reduced $U = 0$ balances, not a microscopic conversion law and not a statement about the full thermal extension.

Differentiation of the other constraint gives

$$HD = s_{\text{ex}} - B'\delta, \quad \mathcal{D} = 3B' + 3\rho_m + 8u_\gamma. \quad (100)$$

Imposing this relation alone does not remove the defect in Eq. (99). With added transfers Q_m, Q_γ and $\dot{\mathcal{R}} = -2H\mathcal{R} + \Xi$,

$$HD = s_{\text{ex}} + Q_m + 2Q_\gamma - B'\delta, \quad (101)$$

$$\dot{\mathcal{C}}_H = P\Xi - Q_m - Q_\gamma - \varepsilon'_z\delta. \quad (102)$$

An alternative birth state must supply these compensating terms through its own physical balances.

C. Growing domains and kinetic input

Nonuniform birth constrains the weighted integral $\Gamma_\partial^{-1} \int q/J_b dA_g = 1/\bar{J}$. Multiple production fronts constrain the sum of their reference contributions; remodeling adds its rate to $\dot{V}_{\text{ref}}$. A receding Core is not automatically an $\mathcal{A}$ -production channel: that requires a specified conversion and reservoir domain.

A law for old material applies to the growing domain only with consistent birth charges and curvature. For a transported scalar X , the surface terms in the derivative of its volume average give

$$\left. \frac{d\bar{X}}{dT} \right|_{\text{birth}} = 3g_\partial(X_b - \bar{X}) \quad (103)$$

for a uniform production surface. Reference matching does not set $\mathcal{R}_b - \bar{\mathcal{R}}$ or the corresponding charge differences. They must vanish or be included when using Eq. (96) for global averages.

If a front is postulated to move at local propagation speed $c_\mathcal{A}$, then in zero-shift material coordinates its velocity per unit T is $q = N_b c_\mathcal{A}$. Writing $q = c_\mathcal{A}$ uses the local cadence convention or $N_b = 1$. This optional kinetic input fixes a speed, not the birth ratio, efficiency, or surface stress.

XIV. FORMATION ENERGY AND THERMODYNAMIC ACCOUNTING

A. Stored density and global reference work

Combine the background and reservoir as

$$\mathcal{H}_A = b_0 I, \quad I = \int_\Omega N dV_g = V\bar{N}, \quad p_A = -b_0 - a_0/N. \quad (104)$$

Add δV at cadence N_b with $\delta V_{\text{ref}} = \delta V/J_b$, holding old fields fixed during this directional variation. Then $\delta I = N_b \delta V$ and $\delta e = (1/V - 1/(V_{\text{ref}} J_b)) \delta V$. Thus

$$\mu_{\text{add}}^{(T)} = \frac{\delta \mathcal{H}_A}{\delta V} = N_b b_0 + a_0(1 - \bar{J}/J_b). \quad (105)$$

The second term changes the existing global reservoir; it is not a density deposited solely in the new material. The reference has the conjugate

$$\left. \frac{\partial \mathcal{H}_A}{\partial V_{\text{ref}}} \right|_{V,I} = -I\varepsilon'_z/V_{\text{ref}}. \quad (106)$$

Matched birth gives $\mu_{\text{add}}^{(T)} = N_b(u_0 + \varepsilon_z)$. A negative marginal cost in a different birth state would describe sectorial energy release, not negative newborn density.

With $\mathcal{L}_T^{\text{mat}} N$ the cadence change followed with old material per unit T , the full transport identity is

$$\begin{aligned} \dot{\mathcal{H}}_A = & \int_\Omega b_0 \mathcal{L}_T^{\text{mat}} N dV_g + \int_\Omega (b_0 N + a_0) \theta dV_g \\ & + \int_{\partial\Omega} (b_0 N_b + a_0) q dA_g - I\varepsilon'_z \frac{\dot{V}_{\text{ref}}}{V_{\text{ref}}}. \end{aligned} \quad (107)$$

The dilation term equals $-\int N p_A \theta dV_g$. Equation (92) combines the surface and reference terms into $\int \mu_{\text{add}}^{(T)} q dA_g$. Cadence work remains explicit.

B. Conversion cost and entropy

For uniform production with $\Gamma_\partial > 0$, after separating old-bulk work, the first law in a common energy and time convention reads

$$\mathcal{P}_\gamma + \mathcal{P}_{\text{mec,in}} = \Gamma_\partial \mu_{\text{add}}^{(T)} + \dot{\mathcal{E}}_\Sigma + \mathcal{P}_{\text{out}}. \quad (108)$$

Here $\mathcal{E}_\Sigma$ is layer energy; $\mathcal{P}_{\text{out}}$ includes outgoing heat, radiation or other products; and $\mathcal{P}_{\text{mec,in}}$ is net work received from other sectors. Internal transfers are counted once, with no external reservoir. The photon cost per produced volume, $U_{\text{form}}^{(T)} = \mathcal{P}_\gamma/\Gamma_\partial$, is

$$U_{\text{form}}^{(T)} = \mu_{\text{add}}^{(T)} + \frac{\dot{\mathcal{E}}_\Sigma + \mathcal{P}_{\text{out}} - \mathcal{P}_{\text{mec,in}}}{\Gamma_\partial}. \quad (109)$$

It has units of energy density but describes a process. Equality with residual density requires additional conditions. In particular, the companion target $U_\partial = P/\ell_\star^2$ [3], in its own energy convention, proves neither $u_{\text{birth}} = P/\ell_\star^2$ nor $u_E = P/\ell_\star^2$. Local energy and energy weighted by N_b must be converted before comparison.

In a common local convention, a conversion control volume satisfies

$$\dot{\mathcal{E}}_{\text{cv}} = \mathcal{P}_{\text{in}} - \mathcal{P}_{\text{out}}, \quad (110)$$

$$\dot{\mathcal{S}}_{\text{cv}} = \mathcal{J}_{S,\text{in}} - \mathcal{J}_{S,\text{out}} + \dot{\mathcal{S}}_{\text{prod}}, \quad \dot{\mathcal{S}}_{\text{prod}} \geq 0. \quad (111)$$

At constant reference temperature $\Theta > 0$, put $F_{\text{cv}} = E_{\text{cv}} - \Theta \mathcal{S}_{\text{cv}}$. Then

$$\begin{aligned} \mathcal{P}_{\text{in}} - \mathcal{P}_{\text{out}} - \Theta(\mathcal{J}_{S,\text{in}} - \mathcal{J}_{S,\text{out}}) - \dot{F}_{\text{cv}} \\ = \Theta \dot{\mathcal{S}}_{\text{prod}} \geq 0. \end{aligned} \quad (112)$$

Identifying stored energy with a free-energy potential requires an entropy law or an explicit athermal assumption. An energy flux alone does not fix its entropy flux. These balances constrain admissible conversion without assigning an unprovided yield or spectrum.

XV. TRANSFER FROM CADENCE TO OBSERVABLES

A. Stationary endpoint redshift

For the RM electromagnetic principal symbol [2], at zero shift,

$$\frac{\omega_T^2}{N^2 c_{\mathcal{E}}^2} = g^{ij} k_i k_j. \quad (113)$$

In a time-independent medium, the frequency conjugate to T is conserved along a ray. For stationary emitters and receivers using the same cadence interface, the local frequency is $\omega_{\text{cad}} = \omega_T/N$. The logarithmic redshift of a specified emitted line is therefore

$$\boxed{\mathcal{Z}_{\text{cad}} \equiv \ln(1 + z_{\text{cad}}) = \ln \frac{N(M)}{N(x_{\text{em}})}}. \quad (114)$$

Stationarity, the local clock interface and the emitted-line convention are necessary premises; the principal symbol alone does not establish transfer in an evolving medium.

With the equal-depth local sampling of Sec. II,

$$\begin{aligned} \mathcal{Z}_{\text{cad}}^-(s, \hat{\mathbf{n}}) &= -s \mathbf{D} \ln N(M) \cdot \hat{\mathbf{n}} + O(s^3) \\ &= -s g_o \cos \theta + O(s^3), \quad g_o = \frac{N'(r_o)}{N(r_o)}. \end{aligned} \quad (115)$$

Here $\cos \theta = \hat{\mathbf{e}}_r \cdot \hat{\mathbf{n}}$. The same leading coefficient holds for a smooth reversible static ray congruence parametrized by spatial arclength. Opposite initial directions have equal even-order displacement, so the next antipodal odd term is cubic.

If the solved field gives $g_o > 0$, redshift increases toward the center, in direction $-\hat{\mathbf{e}}_r$; if $g_o < 0$, this sign reverses. A positive Core junction datum alone does not fix g_o in the global problem. The complete matching solution does.

The carrier here is the potential $\ln N$, not an independently derived direction-independent kernel $K_{\mathcal{E}}(r)$. Equivalently, Eq. (114) integrates the directional derivative $-\dot{\gamma}^i D_i \ln N$ from observer to emitter. Its order- s dipole differs from the order- s^2 law for an integrated scalar in Eq. (5). If $g_o = 0$, the displayed redshift dipole vanishes but an order- s^3 odd term may remain.

B. Counts and temperature: what is not fixed by the axis

A flux limit requires a luminosity function, intrinsic source density, spectral response, optical area mapping and evolution/selection model. The eikonal does not supply these. A redshift-induced dimming contribution may reduce counts in one direction under additional fixed-source assumptions, but Eq. (7) depends on the full effective selection gradient. No universal sign or coefficient linking that gradient to g_o follows here.

A useful exact counterexample concerns radiation temperature. Consider a centered static spherical emitting surface at $r = R_s$, with uniform emitted blackbody temperature T_s , and collisionless propagation preserving photon phase-space occupation, such that every observed direction terminates on that surface. Equation (114) gives

$$T_{\text{obs}}(\hat{\mathbf{n}}) = T_s \frac{N(R_s)}{N(M)}, \quad (116)$$

independent of sky direction, even for an off-center observer in a radial field. Lensing preserves this uniform surface brightness. Thus $DN(M) \neq 0$ does not by itself force a temperature dipole. No origin model for the cosmic microwave background is assumed in this example.

Axisymmetry survives both these qualifications. Parallel or antiparallel axes do not imply identical signs or nonzero amplitudes across different observables.

C. A local redshift-slope diagnostic

Suppose a separately derived contribution has leading isotropic short-distance slope K_M , so that $\mathcal{Z} = K_M s - s g_o \cos \theta + O(s^2)$. The following redshift-slope proxy is defined on a single spatial slice; it is not identified with an observational Hubble-parameter estimator:

$$H_{\text{app}}(s, \hat{\mathbf{n}}) \equiv c_{\mathcal{E}} \frac{\mathcal{Z}(s, \hat{\mathbf{n}})}{s}. \quad (117)$$

Its leading odd part is

$$H_{\text{app}}^- = -c_{\mathcal{E}} g_o \cos \theta + O(s). \quad (118)$$

It is independent of depth only at this leading local order. If the additional contribution is exactly isotropic, the odd remainder improves to $O(s^2)$. If $K_M \neq 0$, its leading relative dipole amplitude is $|g_o/K_M|$. This is not automatically an estimator of the measured Hubble parameter or the homogeneous RM rate H : a luminosity-distance law, source motions and time-dependent propagation must first be derived. No supernova bound is converted into a Core mass here.

XVI. PREDICTIVE USE AND MECHANICAL DOMAIN

The inference order is constitutive and boundary physics, a compatible geometry and cadence solution, the observer's gradient, a specified propagation and selection law, and only then comparison with observations. The mean $\bar{N}$ is calculated on the actual reservoir domain throughout. A scalar screened profile on prescribed curvature is not promoted to a static coupled solution in conflict with Eq. (45).

The symmetry result concerns nonzero dipoles independently attributed to the same spherical structure. Significant noncollinearity rejects that common origin; a zero dipole is compatible with it. Amplitude and sign tests concern a branch with fixed constitutive and matching data. Equations (71) and (72) expose that dependence. No historical density anchor or observed direction is used to select their sign.

The dynamical equations, birth theorem, formation balance, and interface identities constitute mechanical compatibility conditions. Matter transport, boundary motion, and global averages must satisfy them in a concrete solution. The stationary redshift formula retains its own stationarity and endpoint assumptions; it does not replace time-dependent propagation.

XVII. CONCLUSION

The global elastic functional produces an exact mean-cadence term in the spatial scalar constraint. The full tensor equation restricts static solutions, while Core mechanics and interface stresses determine transmitted flux. Exact regular interiors exhibit positive, zero, and negative fluxes at identical density and radius through different boundary pressures.

Boundary production couples reference bookkeeping to energy. Within the reduced $U = 0$ homogeneous branch and its declared material-radiative balances, preserving both constraints gives $\dot{C}_H = -\epsilon'_z \delta$ and selects matched birth for a single uniform front without remodeling. The same energy variation separates newborn storage from changes of the whole reservoir. Formation cost additionally includes layer storage, outgoing products, and work exchanged with other sectors.

The radial dynamics retain the trace-free kinetic contribution, the canonically normalized momentum current, and independent principal stresses. Their traction contains $-S_b \mathcal{B}_{r,b}/(3c_{\mathbb{E}}^2)$ and reduces to static matching when the shear and its rate vanish. These results give explicit compatibility conditions linking a finite Core, a produced bulk, and its global elastic state.

For a stationary matched branch the local observable connection is

$$\mathcal{Z}_{\text{cad}}^-(s, \hat{\mathbf{n}}) = -s \frac{N'(r_o)}{N(r_o)} \hat{\mathbf{e}}_r \cdot \hat{\mathbf{n}} + O(s^3).$$

Spherical symmetry fixes the axis; the coupled mechanical solution fixes the gradient and hence the sign and leading amplitude of this cadence contribution.

Appendix A: Two-boundary scalar solution on constant curvature

Take $r_a < r < r_b$, $S_\kappa > 0$, and, when $\kappa > 0$, $r_b < \pi/\sqrt{\kappa}$. Set $q^2 = \lambda_z^{-2} - \kappa$ and define

$$W_q(t) = \begin{cases} \sinh(qt)/q, & q^2 > 0, \\ t, & q^2 = 0, \\ \sin(\beta t)/\beta, & q^2 = -\beta^2 < 0. \end{cases} \quad (\text{A1})$$

For Dirichlet values n_a, n_b , the unique solution of Eq. (38) is

$$n(r) = \frac{1}{S_\kappa(r)W_q(r_b - r_a)} \left[S_\kappa(r_a)n_a W_q(r_b - r) + S_\kappa(r_b)n_b W_q(r - r_a) \right]. \quad (\text{A2})$$

For $\lambda_z^{-2} > 0$ and the stated normal-region restriction the denominator is nonzero. In the trigonometric case, $\beta < \sqrt{\kappa}$ and $\beta(r_b - r_a) < \pi$. Both boundaries enter explicitly. Values defined by $n_i = N_i - \bar{N}$ must subsequently obey the mean condition on the actual reservoir domain; this auxiliary Dirichlet problem does not independently fix that mean.

Appendix B: Independent radial variations

At zero shift, integrate the curvature term by parts in the interior. Up to the angular factor 4π and an endpoint term, the reduced Lagrangian is

$$L_g = \frac{2P}{3c_{\mathbb{E}}^2} \frac{AS^2}{N} \zeta^2 + P \left(\frac{4N'SS'}{A} + \frac{2N(S')^2}{A} + 2NA \right). \quad (\text{B1})$$

The discarded derivative is $-4P(NSS'/A)'$. It does not affect compactly supported bulk variations, but cannot be discarded to derive boundary dynamics.

Let $\mathcal{E}_A = \partial L/\partial A - \partial_T(\partial L/\partial \dot{A}) - \partial_r(\partial L/\partial A')$, and similarly for S . Vary before setting $A = 1$, $A' = 0$ on a slice. The kinetic terms are

$$\mathcal{E}_A^{\text{kin}} = -\frac{4PNS^2}{3c_{\mathbb{E}}^2} (D_\tau \vartheta + \vartheta^2/2 + 3\vartheta H_t), \quad (\text{B2})$$

$$\mathcal{E}_S^{\text{kin}} = \frac{4PNS}{3c_{\mathbb{E}}^2} (D_\tau \vartheta + 2\vartheta^2 + 3\vartheta H_t). \quad (\text{B3})$$

The curvature contributions are

$$\mathcal{E}_A^{\mathcal{R}} = 2PN(1 - y^2) - 4PN'Sy, \quad (\text{B4})$$

$$\mathcal{E}_S^{\mathcal{R}} = -4P(SN'' + yN' + NS''). \quad (\text{B5})$$

The matter interface adds NS^2p_r and $2NASp_t$. Dividing the resulting equations by $-4PNS^2$ and $-4PS$ yields Eqs. (81) and (82). Direct Euler–Lagrange variation of the unreduced curvature expression with second radial derivatives yields the same bulk equations.

In their sum the terms $D_\tau\vartheta + 3\vartheta H_t$ cancel. The remaining quadratic contribution is $N\vartheta^2/(2c_{\mathcal{E}}^2)$. Combining it with Eq. (80) produces $2/(3c_{\mathcal{E}}^2)$ in Eq. (84).

CALCULATION METHODS AND AVAILABILITY

No empirical sky data were analyzed or used for parameter selection. The analytical derivations are included in this article. The accompanying source archive provides

the LaTeX manuscript and reproducible checks of regular interiors, reference and energy transport, formation identities, radial constraint combinations, traction signs, and homogeneous compatibility. The numerical examples use synthetic dimensionless inputs. These checks verify the displayed identities, not the existence or stability of a complete cosmological solution.

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TABLE I. Principal results and their hypotheses. This map abbreviates the assumptions; the cited equations and their surrounding text specify the full branch, boundary and regularity conditions. None of these identities alone constructs a cosmological solution.

Result	Hypotheses and domain	Equations
Exact sky axis	The complete configuration and observation law respect spherical symmetry; the observer has no transverse velocity. A nonzero dipole is collinear with OM .	(2)
Local odd amplitudes	Smooth scalar field or effective density, small geodesic depth and a normal neighborhood. The relative count amplitude requires $\nu(M) > 0$.	(5), (7)
Global mean-cadence term	The GEP functional is adopted; the reservoir domain and positive reference volume are fixed during variation. Canonical kinetic terms are retained in the scalar constraint.	(14), (19)
Screening-length lower bound	Positive ε'_z , nonnegative homogeneous component densities and the homogeneous constraints; a is the curvature radius of their constant-curvature representative.	(31), (32)
Static zero-gradient obstruction	Connected static region, zero trace-free momentum, isotropic stresses, constant $\mathcal{R}$ and $\alpha_z > 0$, $\mathcal{Q}_{loc} = 0$, and $\alpha_z + \mathcal{R}/2 > 0$.	(45)
Core reaction and active sign	Constrained saturation is varied before substitution. Static isotropic mechanical equilibrium, constant u_E , no interior conversion or additional volume force, and positive cadence are assumed.	(48), (50), (53)
Exact interior and loaded sign tests	Regular constant-density static interior in the minimal branch, $y_b > 0$ and $N > 0$. Pressure, reservoir occupation and junction data remain independent mechanical inputs.	(58)–(61); (71), (72)
Radial dynamics and traction	Minimal action, canonical tensor density and source convention, independent p_r, p_t . Variation precedes the instantaneous proper-distance gauge; the inner bulk normal is $-\partial_r$.	(79)–(85)
Flux and layer identities	The instantaneous source is taken from the actual interior branch. The displayed layer jumps additionally require a static layer, common P and no radial delta stress.	(86)–(88)
Matched reference birth	Reduced $U = 0$ homogeneous branch; both constraints are preserved with the declared material and curvature balances and $\varepsilon'_z > 0$. Uniform nonzero production without reference remodeling then selects $J_b = \bar{J}$. The enlarged thermal branch requires separate rederivation if imported here.	(94), (99)
Formation-cost separation	GEP energy bookkeeping and reference transport; a common energy/time convention. Conversion cost additionally accounts for layer storage, outgoing products and received mechanical work.	(105), (107), (109)
Cadence redshift and spatial proxy	Stationary propagation with the stated endpoint calibration; smooth local fields for the short-distance expansion. Counts and temperature require their own transfer and source laws.	(114)–(118)